SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

BNL Nuclear Physics Seminar

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COLLINS FRAGMENTATION FUNCTION WITHIN NJL-JET MODEL

Hrayr Matevosyan

Collaborators:
A.W.Thomas, W. Bentz & I.Cloet

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

THE UNIVERSITY of ADELAIDE

SUB CRUCE LUMEN

OUTLOOK

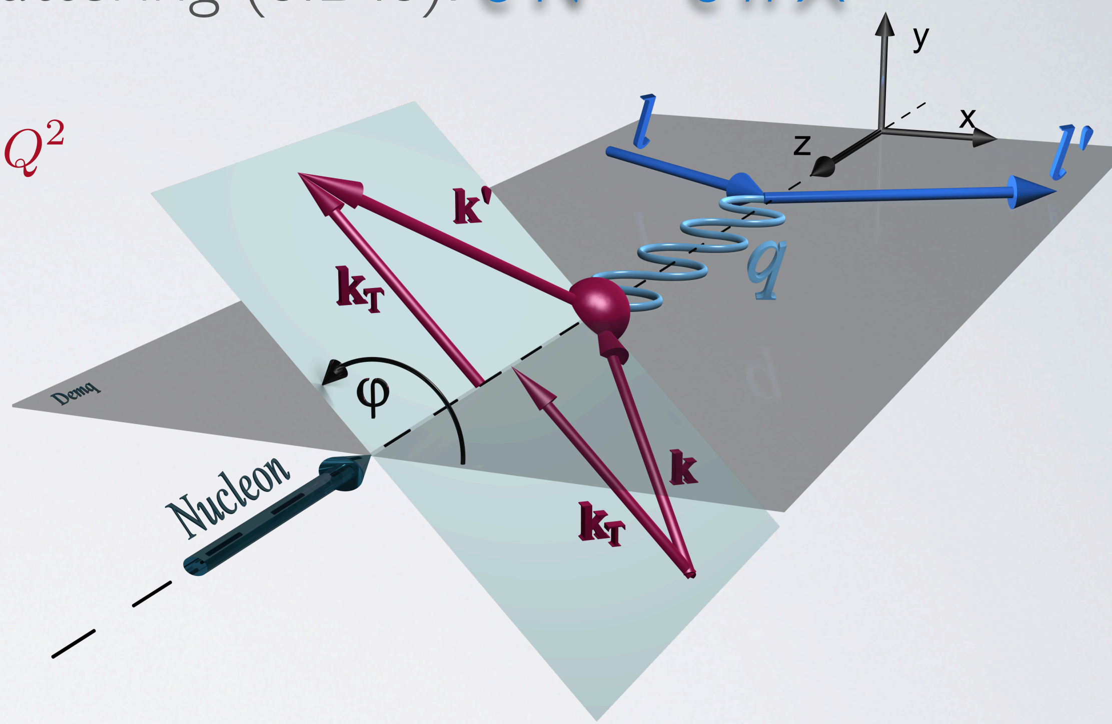
- Motivation.
- Short Overview of the Nambu--Jona-Lasinio (NJL) - jet model and Monte-Carlo approach:
  - Transverse Momentum Dependent (TMD) Fragmentation Functions (FF) and Parton Distribution Functions (PDF).
  - Hadron Transverse Momenta in Semi-Inclusive Deep Inelastic Scattering (SIDIS).
- ***Collins fragmentation functions.***
- ***Higher Order Collins Modulations from quark-jet Framework.***
- Conclusions.

EXPLORING HADRON STRUCTURE

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): $e\,N \rightarrow e\,h\,X$

- Cross-section factorizes: $P_T^2 \ll Q^2$



$$\boxed{\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}}$$

Distribution

$$\frac{d\sigma^{lN\rightarrow l'hX}}{dxdQ^2dzd^2P_T} = \sum_q f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\rightarrow lq} \otimes D_q^h(z, P_\perp^2, Q^2)$$

Fragmentation

- Access to nucleon's transverse structure.
- NJL provides microscopic description of TMD PDFs and FFs!

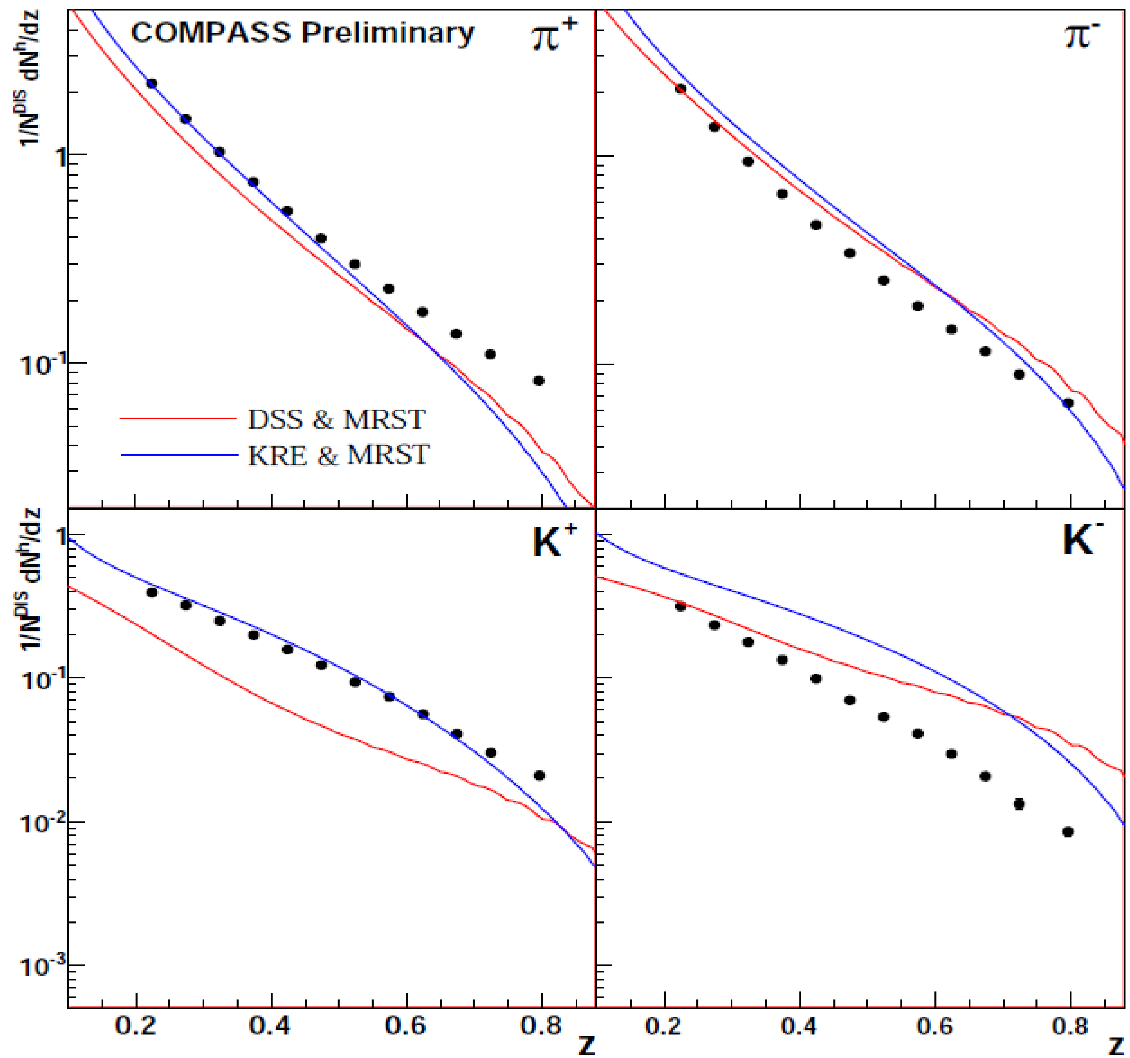
Unfavored FFs NOT well known!

From talk by Celso Franco for COMPASS at CIPANP 2012

$$\frac{dM^h(x,z,Q^2)}{dz} = \frac{\sum_q f_1^q(x,Q^2) D_q^h(z,Q^2)}{\sum_q f_1^q(x,Q^2)}$$

Hadron Multiplicities (Preliminary)

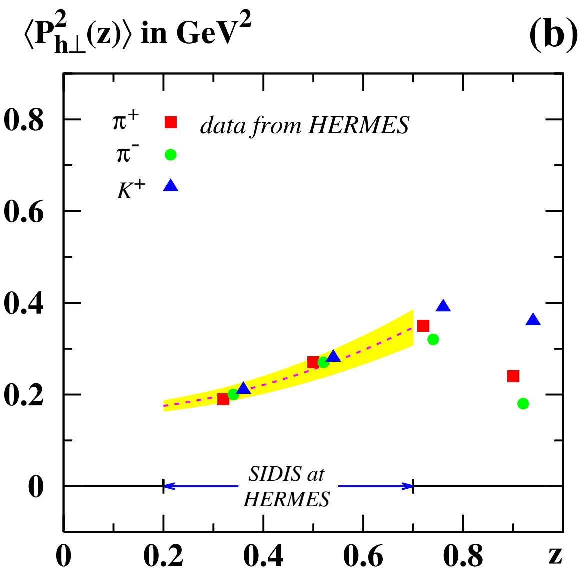
^{6}LiD



AVERAGE TRANSVERSE MOMENTA

P. Schweitzer et al., Phys.Rev. D81, 094019 (2010).

$$\langle k_T^2 \rangle \equiv \frac{\int d^2\mathbf{k_T}\; k_T^2 f(x,k_T^2)}{\int d^2\mathbf{k_T}\; f(x,k_T^2)} \qquad \langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp\; P_\perp^2 D(z,P_\perp^2)}{\int d^2\mathbf{P}_\perp\; D(z,P_\perp^2)}$$



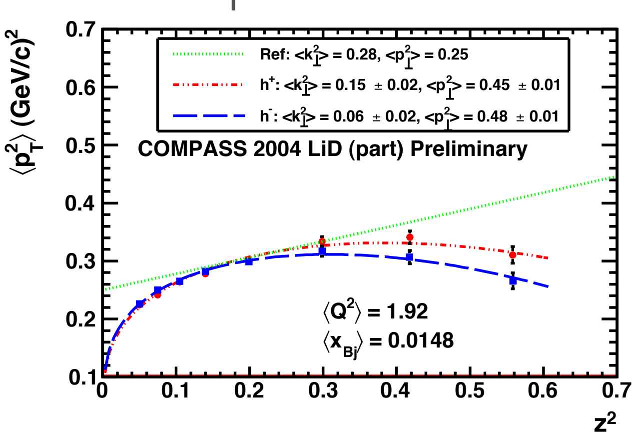
Using Gaussian Ansatz and:

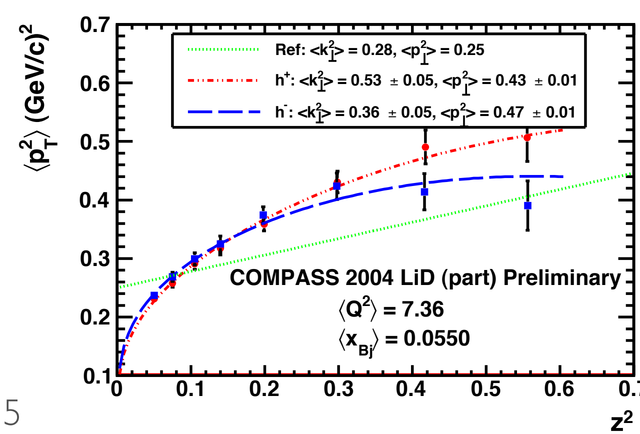
$$D(z,P_\perp^2) = D(z)e^{-P_\perp^2/\langle P_\perp^2\rangle}/\pi\langle P_\perp^2\rangle$$

$$\langle P_T^2\rangle = \langle P_\perp^2\rangle + z^2\langle k_T^2\rangle$$

$$\boxed{\begin{aligned}\langle k_T^2\rangle &= (0.38\pm0.06)\ \mathrm{GeV}^2\\ \langle P_\perp^2\rangle &= (0.16\pm0.01)\ \mathrm{GeV}^2\end{aligned}}$$

Non-trivial z dependence from COMPASS: **Rajotte arXiv:1008.5125**





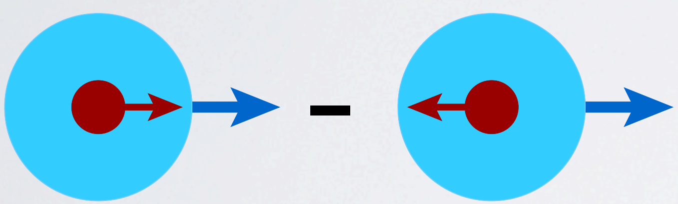
NUCLEON PARTON DISTRIBUTION FUNCTIONS

- ***The momentum Distributions of quarks in nucleon can be non-isotropic once the polarization of the nucleon and the quarks is considered.***

- *Unpolarized* quark in *Unpolarized* nucleon.

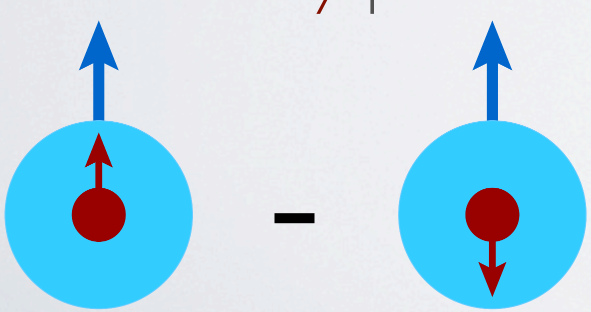
$$f_1^q(x, k_\perp^2)$$

- *Longitudinally* polarized quark in *Longitudinally* polarized nucleon.



$$g_{1L}^q(x, k_\perp^2)$$

- *Transversely* polarized quark in *Transversely* polarized nucleon.



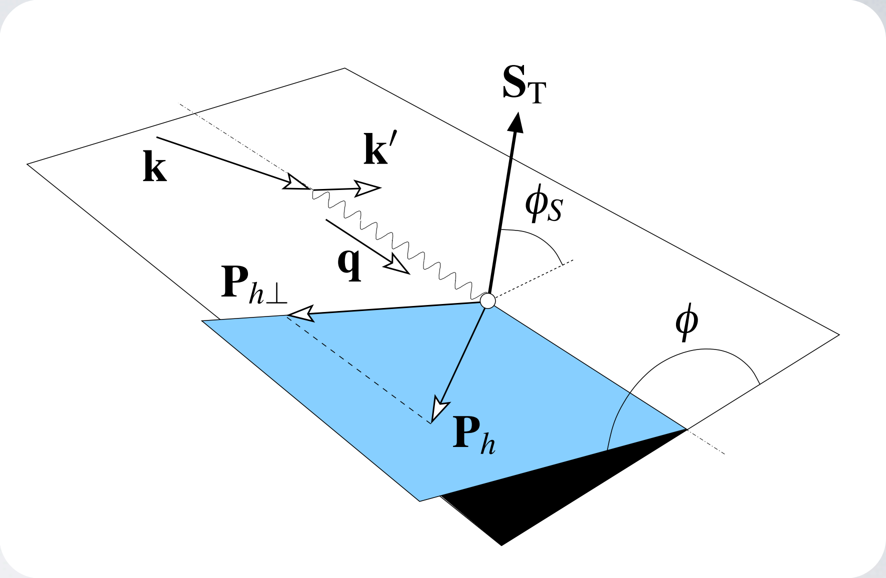
$$h_{1T}^q(x, k_\perp^2)$$

Chiral-odd: Suppressed in Inclusive DIS

SIDIS POLARIZED CROSS-SECTION

A. Bacchetta, JHEP08, 023 (2008).

- **For polarized SIDIS cross-section there are *18 terms* in leading twist expansion:**



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + .. \right]$$

Sivers Effect

Collins Effect

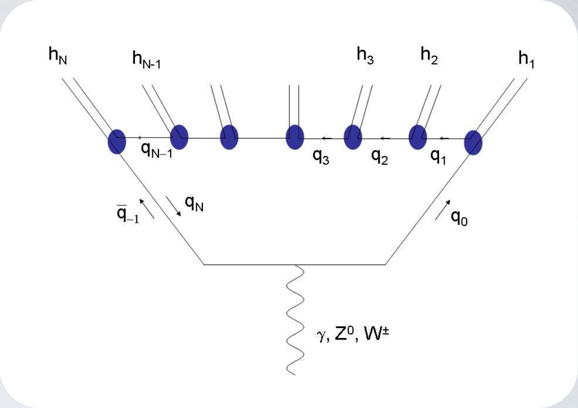
- Extract the specific harmonics:

$$F_{UU} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

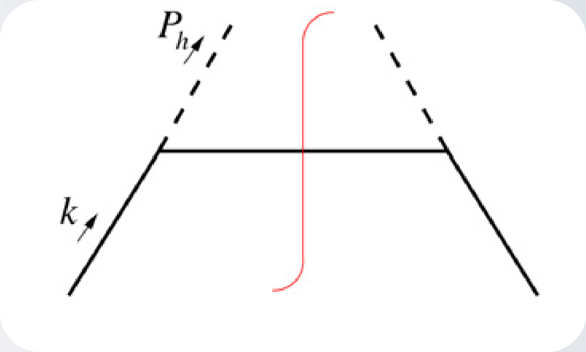
- NEED Collins Function to access the Transversity from SIDIS!

MODELS FOR FRAGMENTATION

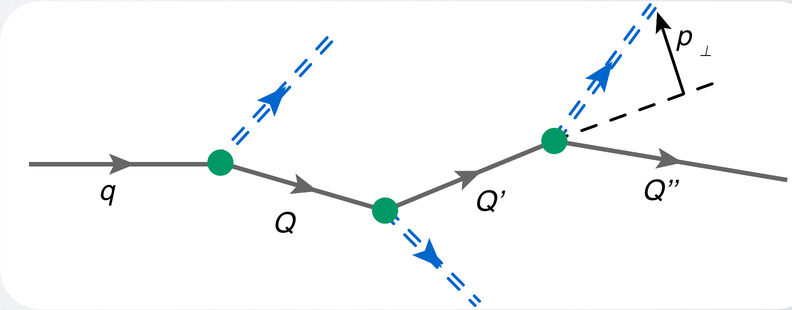
- Lund *String* Model
  - *Very Successful* implementation in ***JETSET, PYTHIA***.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - *Quark model calculations with empirical form factors.*
  - *No unfavored fragmentations.*
  - *Need to tune parameters for small z dependence.*



- *NJL-jet Model*
  - ***Multi-hadron*** *emission framework with effective quark model input.*
  - ***Monte-Carlo framework*** *allows flexibility in including the transverse momentum, spin effects, etc.*



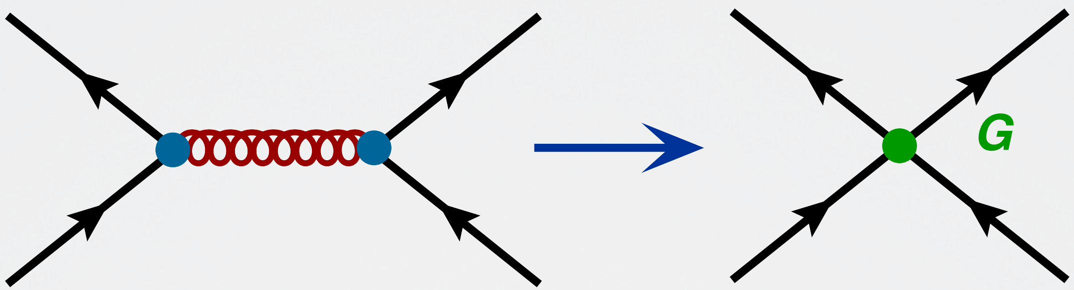
MOTIVATION

- Providing guidance based on a sophisticated model for applications to problems where phenomenology is difficult/inadequate.
- Unfavored fragmentation functions from the model that goes beyond a single hadron emission approximation.
- NO model parameters fitted to fragmentation data!
- Automatically satisfies the sum rules.
- TMD fragmentations in the same model where structure functions were calculated: Full description of cross-sections.
- Experimental hints at similar size and opposite sign of 1/2 moments of favored and unfavored Collins functions. A firm theoretical explanation for this effect has yet to be given.

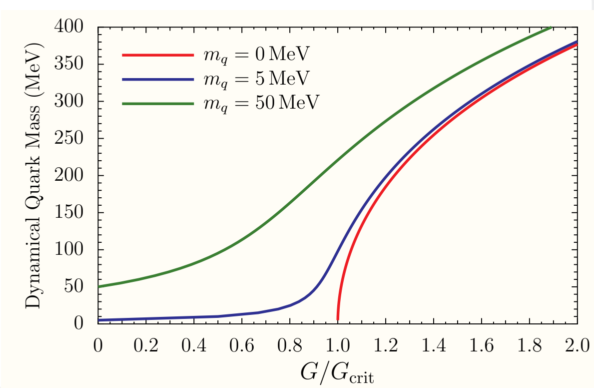
NAMBU--JONA-LASINIO MODEL

Effective Quark model of QCD

- Effective Quark Lagrangian $\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$

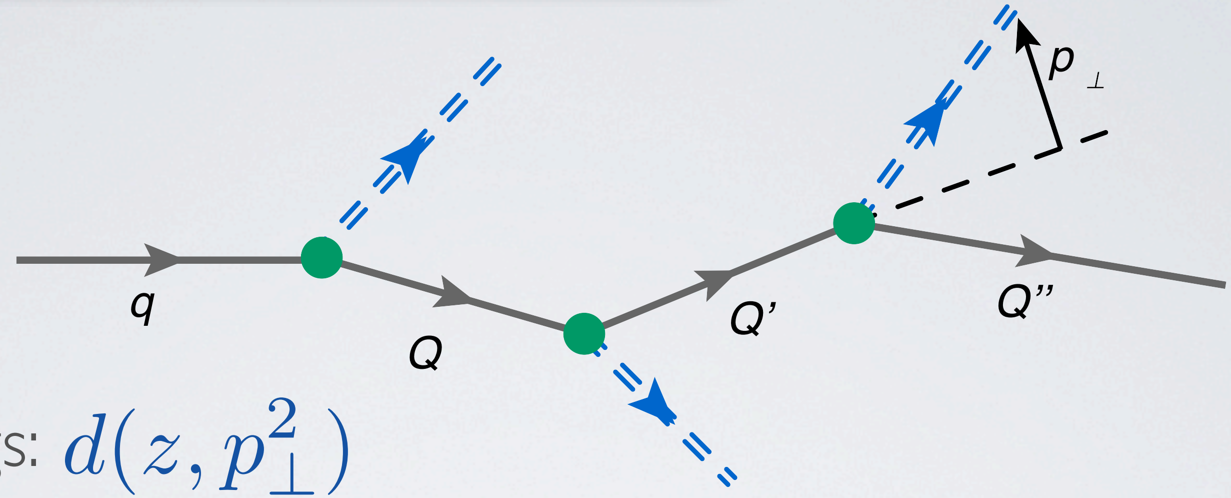


- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.
- Dynamically Generated Quark Mass from GAP Eqn.
- Excellent description of nucleon structure along with the medium modifications, etc.



TMD FRAGMENTATION FUNCTIONS FROM NJL-JET

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

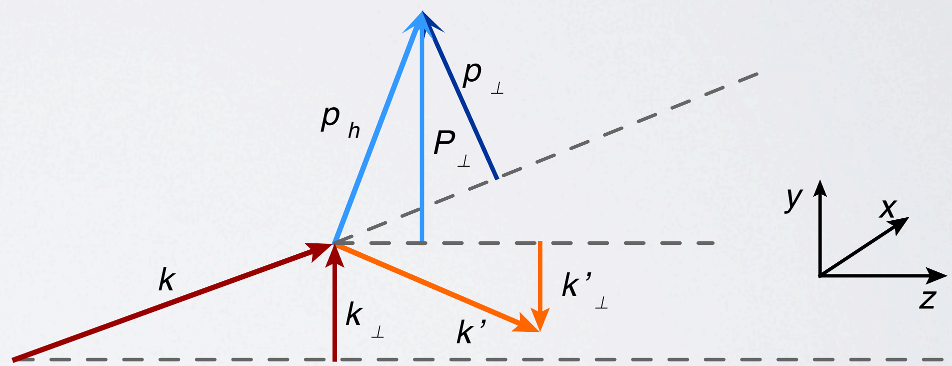


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



- Calculate the Number Density

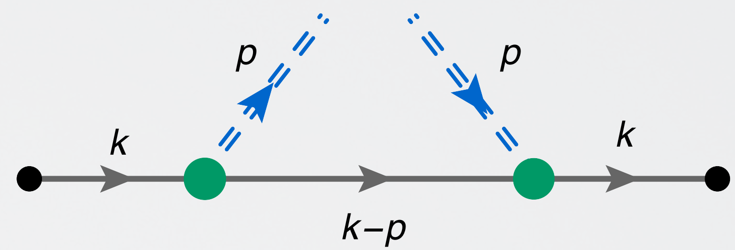
$$D_q^h(z, P_\perp^2)\Delta z\, \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

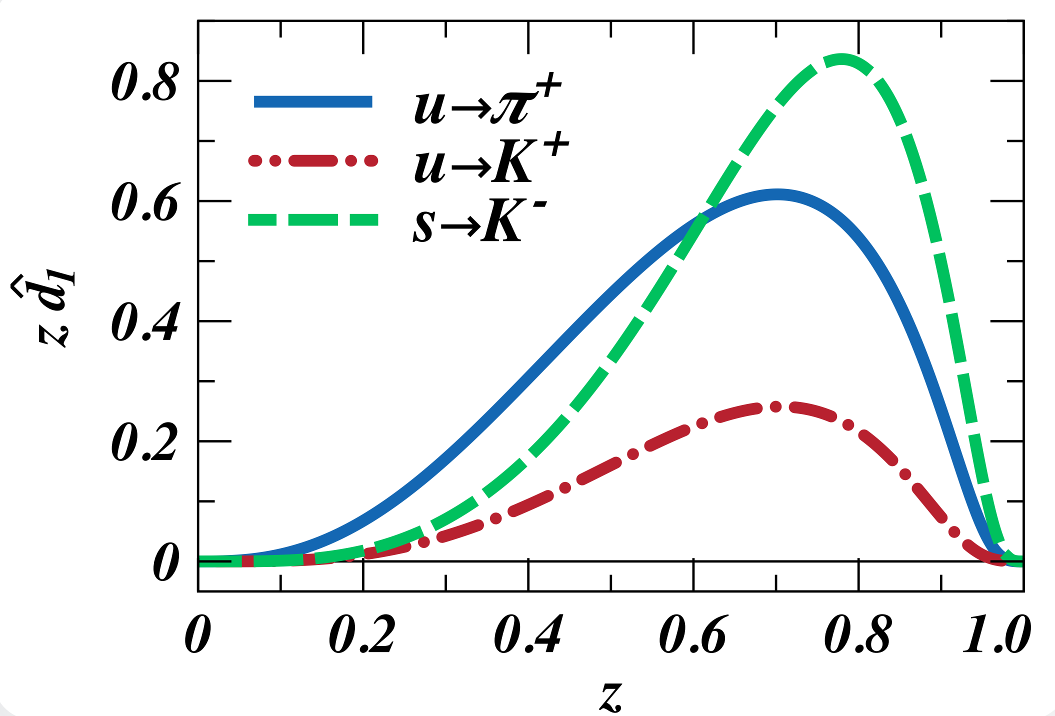
NJL-JET: ELEMENTARY SPLITTINGS

$$\Delta_{ij}(z,p_\perp) = \frac{1}{2N_c\, z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{ip\cdot\xi} \times \langle 0|\mathcal{U}_{(\infty,\xi)}\,\psi_i(\xi)|h,X\rangle_{\text{out out}}\langle h,X|\bar\psi_j(0)\,\mathcal{U}_{(0,\infty)}|0\rangle\Big|_{\xi^-=0}$$

- One-quark truncation of the wavefunction: $d_1^{h/q}(z) : q \to Qh$

$$d_1^{h/q}(z,p_\perp^2) = \frac{1}{2}\mathrm{Tr}\left[\Delta_0(z,p_\perp^2)\gamma^+\right]$$





Drell-Levy-Yan (DLY) Relation

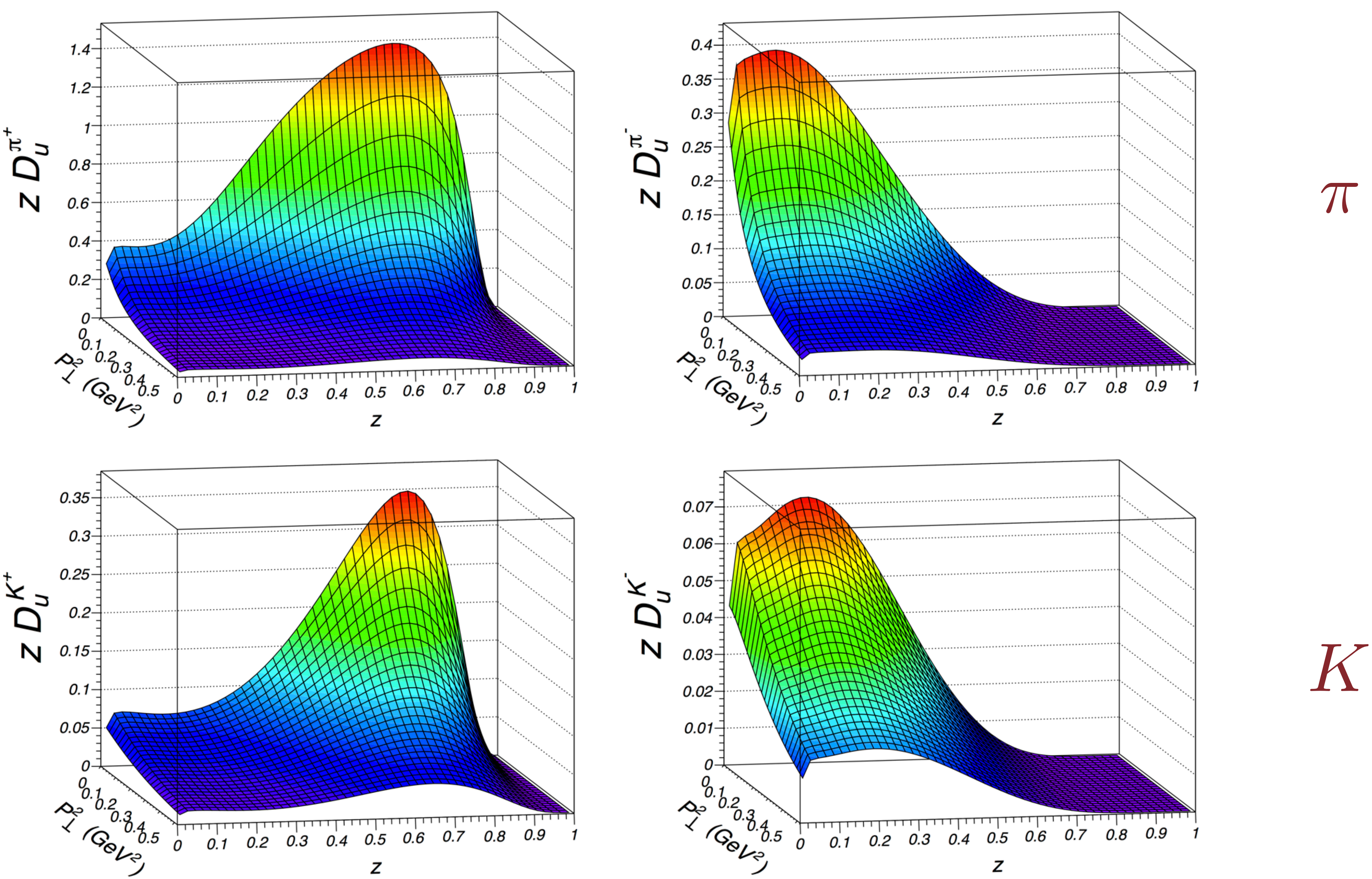
$$d_1^{h/q}(z) = C_q^h\, z\, f_q^h\left(x = \frac{1}{z}\right)$$

$$C_q^h = \frac{(-1)^{2(s_q+s_h)+1}}{d_q}$$

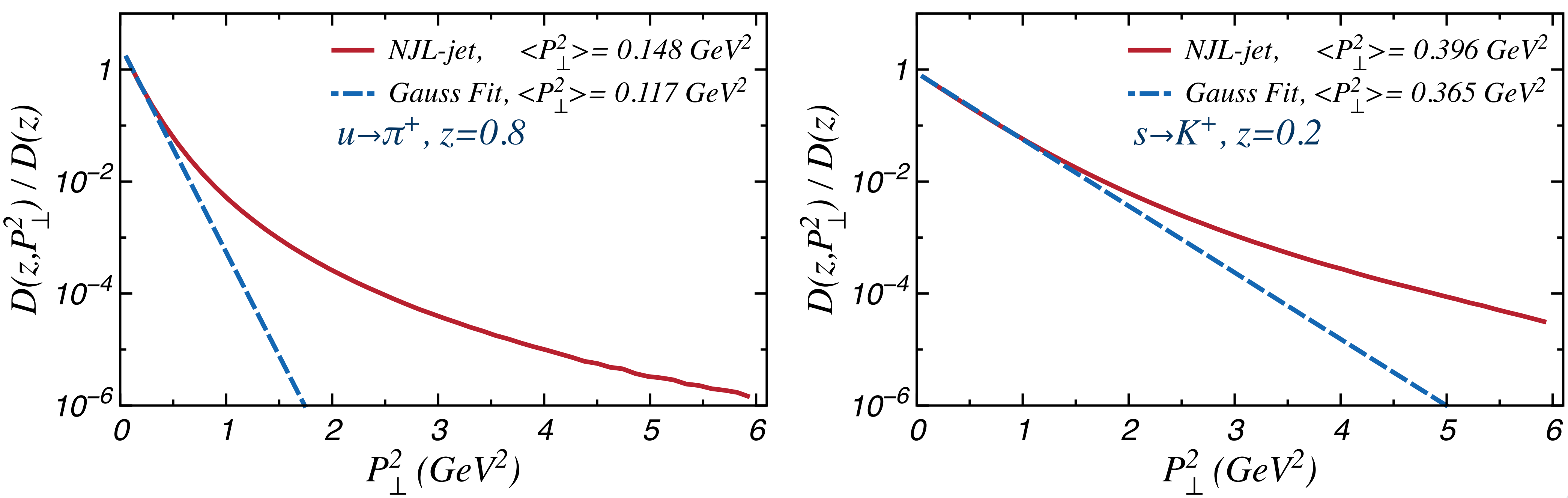
TMD FRAGMENTATION FUNCTIONS

FAVORED

- UNFAVORED

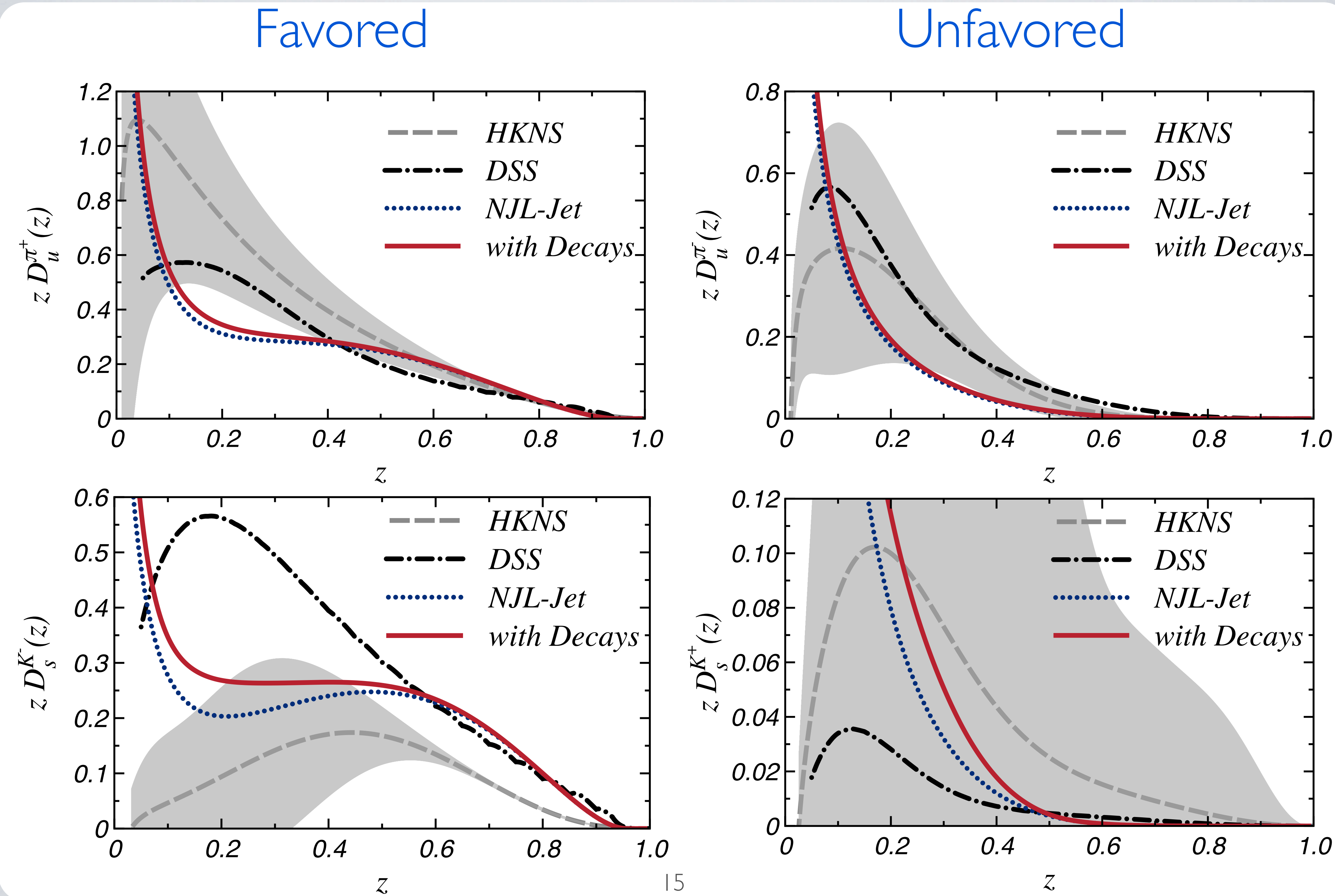


COMPARISON WITH GAUSSIAN ANSATZ

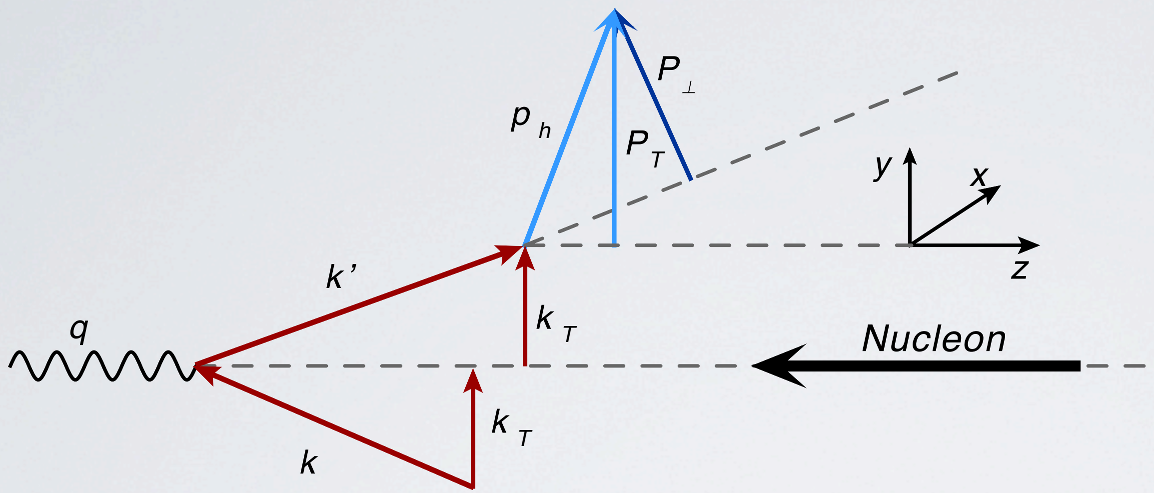


- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z)\dfrac{e^{-P_\perp^2/\langle P_\perp^2\rangle}}{\pi\langle P_\perp^2\rangle}$

Results with vector mesons, N-Nbar: $Q^2 = 4\ \mathrm{GeV}^2$

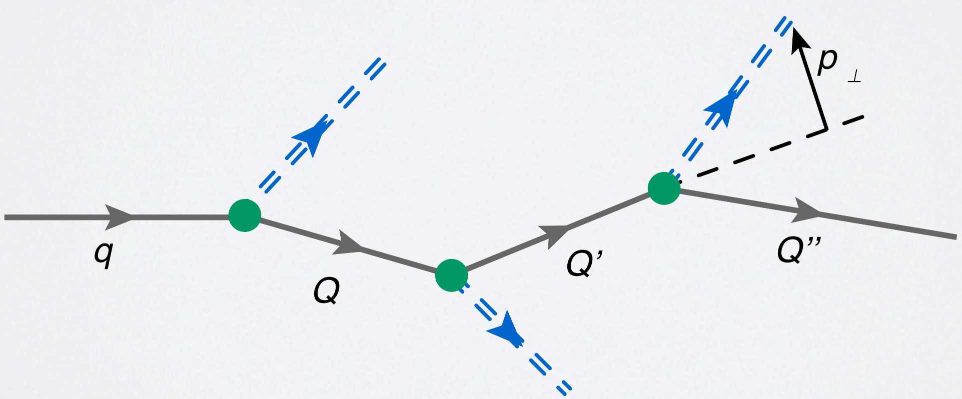


THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$

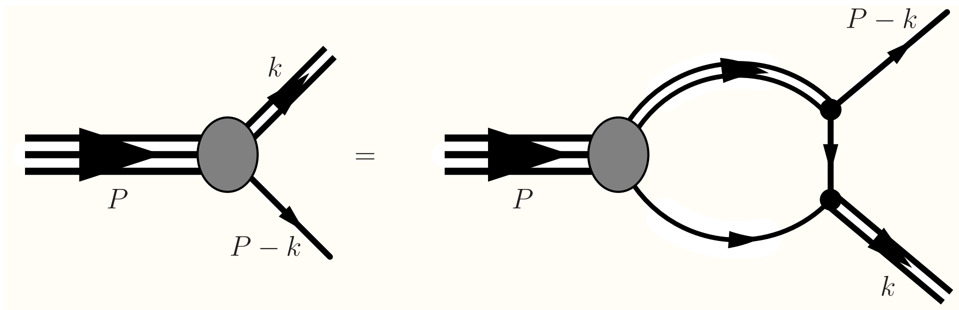
- Use TMD quark distribution functions from the NJL model .
- Use Quark-jet hadronization model and NJL splittings.



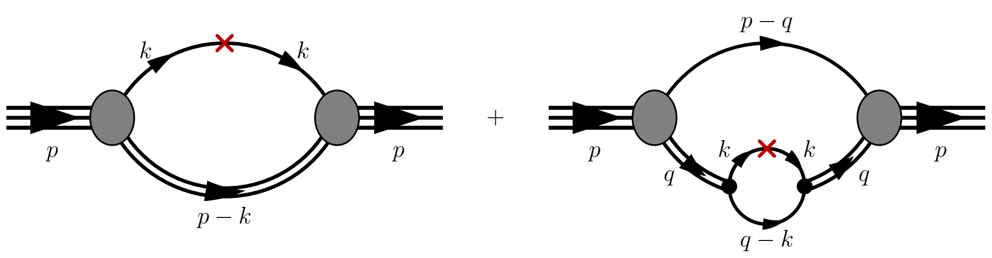
- Evaluate the cross-section using MC simulation.

NJL: NUCLEON PDFS

- Quark-diquark description of Nucleon using relativistic Faddeev approach



- PDFs from Feynman diagrams

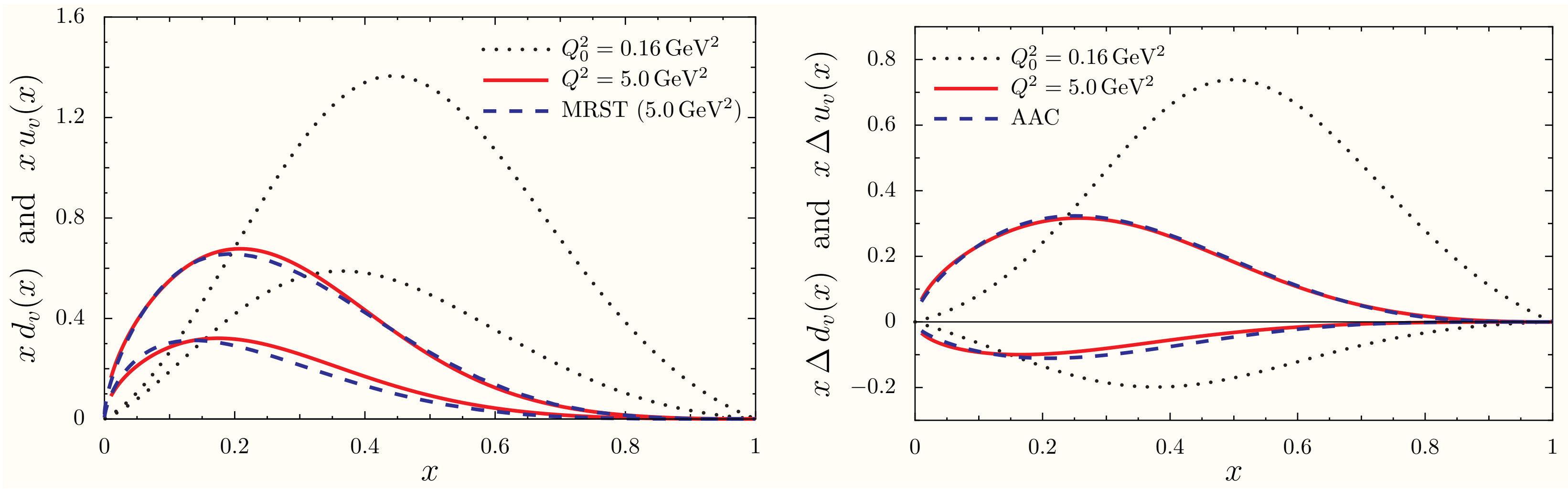


$$\mathcal{Q}(x,\boldsymbol{k_T}) = p^+ \int \frac{d\xi^- d\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ix\,p^+\,\xi^-}\, e^{-i\,\boldsymbol{k_T}\cdot\boldsymbol{\xi_T}}\, \langle N,S\,|\bar{\psi}_q(0)\,\gamma^+\,\mathcal{W}(\xi)\,\psi_q(\xi^-,\xi_T)|\,N,S\rangle\Big|_{\xi^+=0}$$

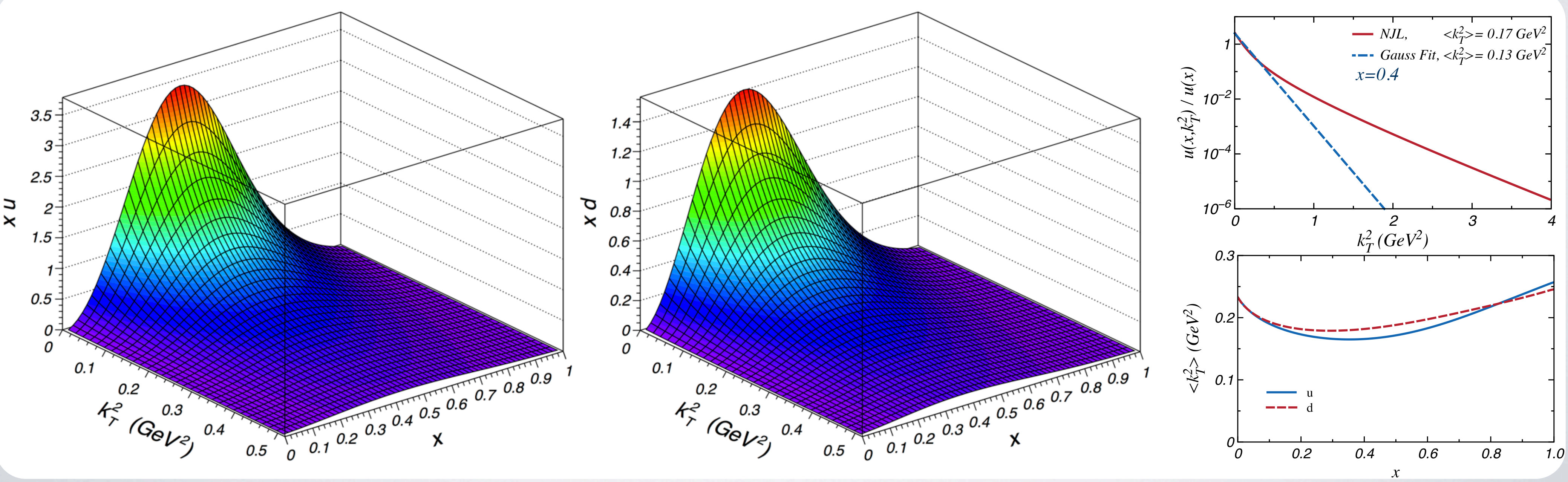
$$\mathcal{Q}(x,\boldsymbol{k_T}) = q(x,k_T^2) - \frac{\varepsilon^{-+ij}\,k_T^i\,S_T^j}{M}\,q_{1T}^{\perp}(x,k_T^2)$$

NJL: NUCLEON PDFS - RESULTS

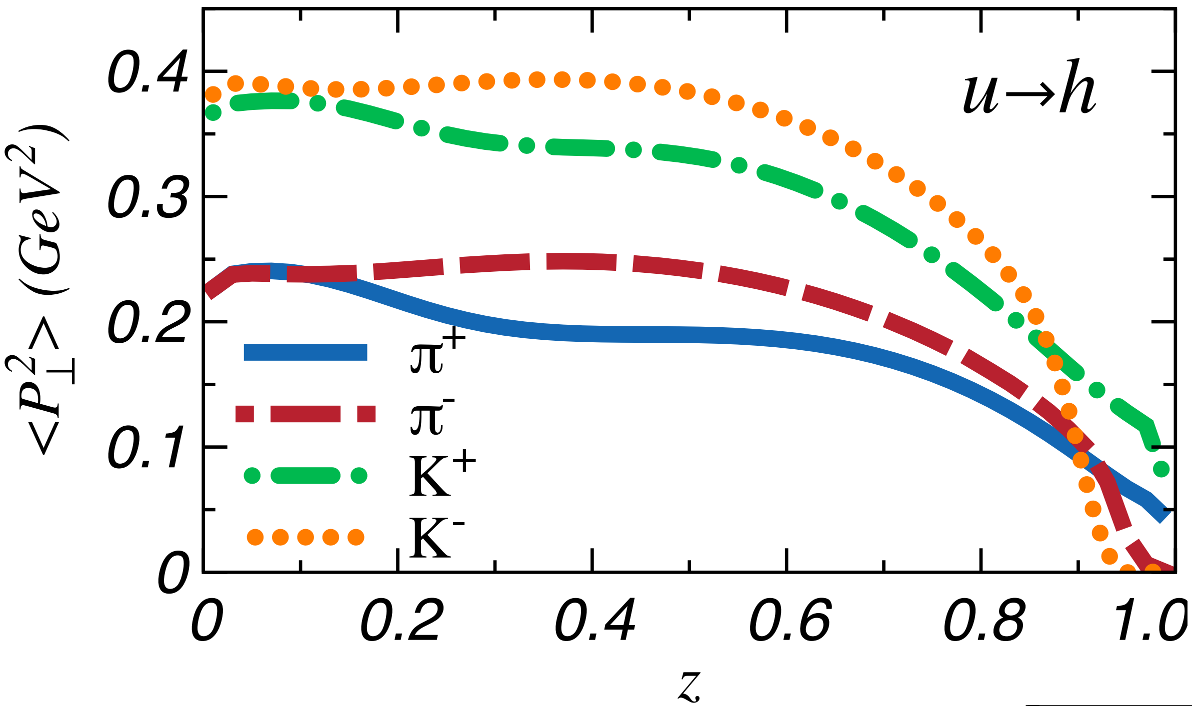
- Integrated PDFs



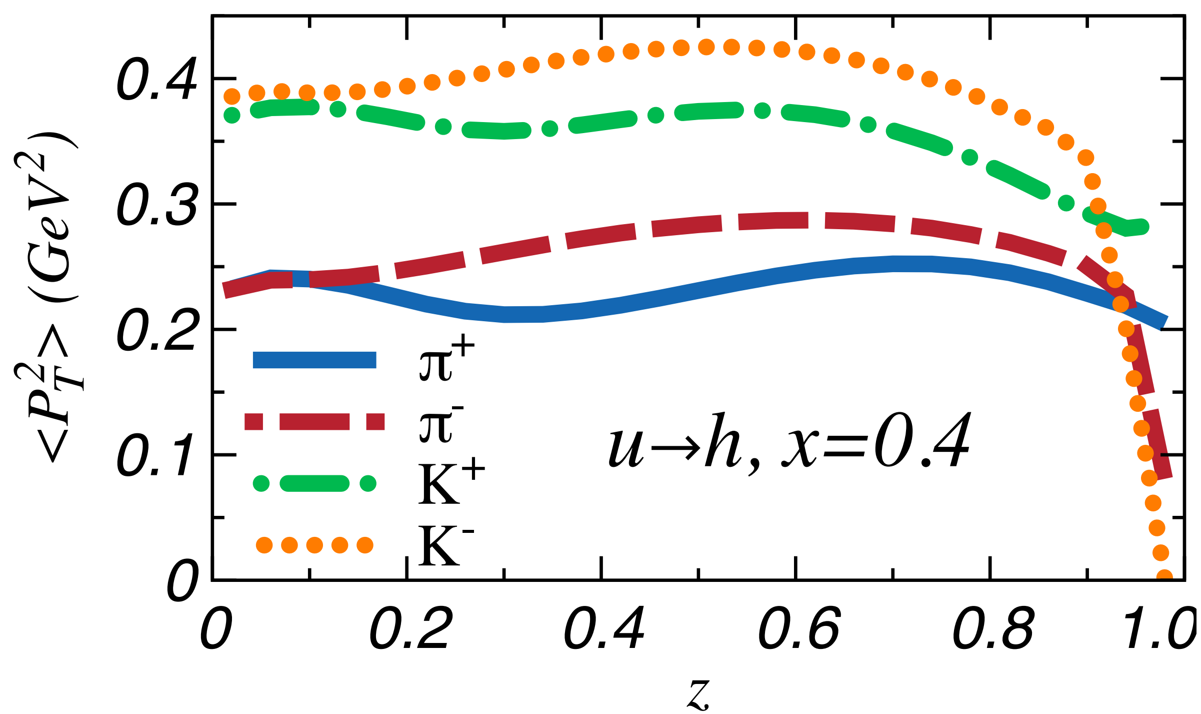
- TMD PDFs



AVERAGE TRANSVERSE MOMENTA VS Z



FRAGMENTATION



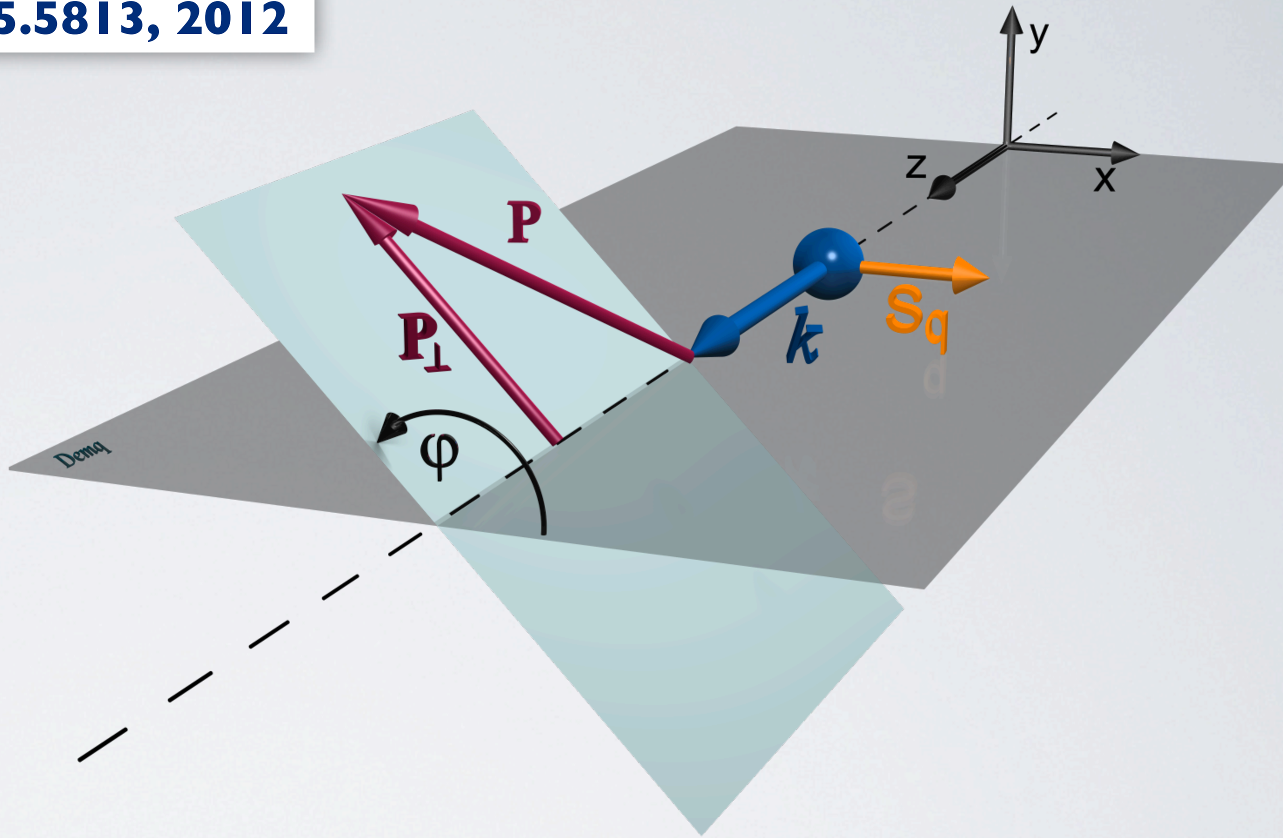
SIDIS

COLLINS FRAGMENTATION FUNCTION

H.M., Thomas, Bentz, arXiv:1205.5813, 2012

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark's Fragmentation Function.



Unpolarized

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

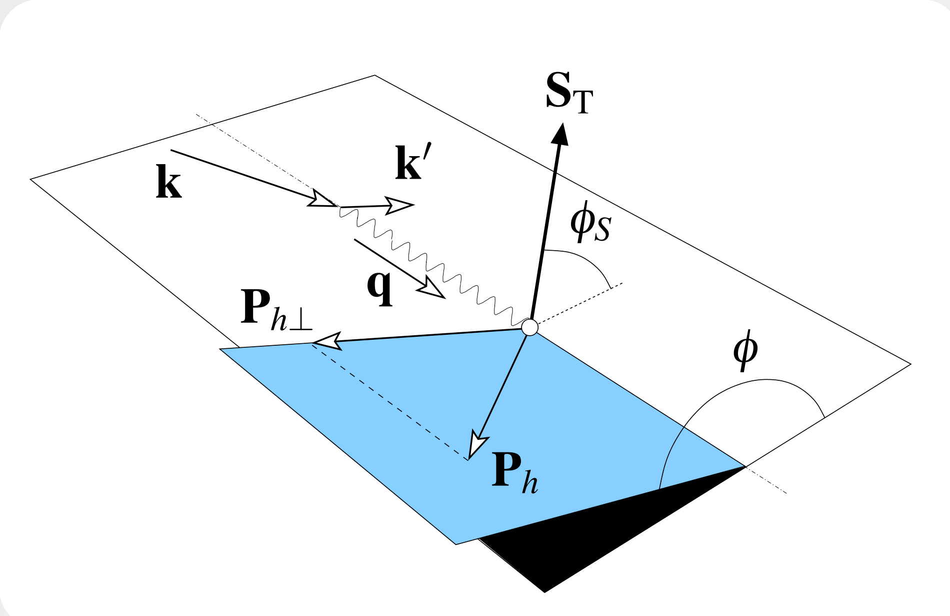
Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

EXPERIMENTAL MEASUREMENTS IN HERMES $l\ \vec{p} \to l'\ h\ X$

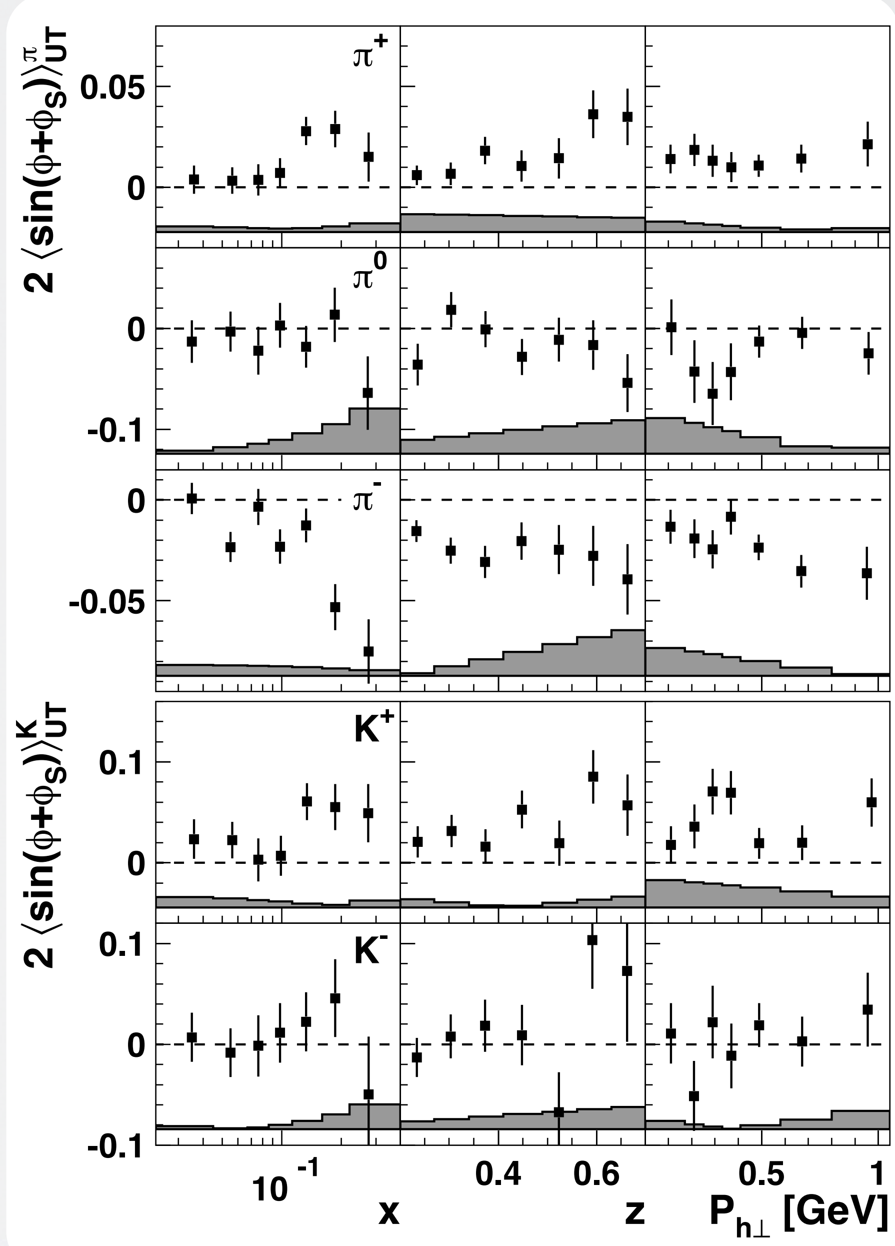
Airapetian et al, Phys.Lett. B693 (2010) 11-16.

- SIDIS with transversely polarized target:



$$\langle \sin(\phi+\phi_S)\rangle^h_{UT} \sim \frac{\mathcal{C}[h_1^q\ H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q\ D_1^{h/q}]}$$

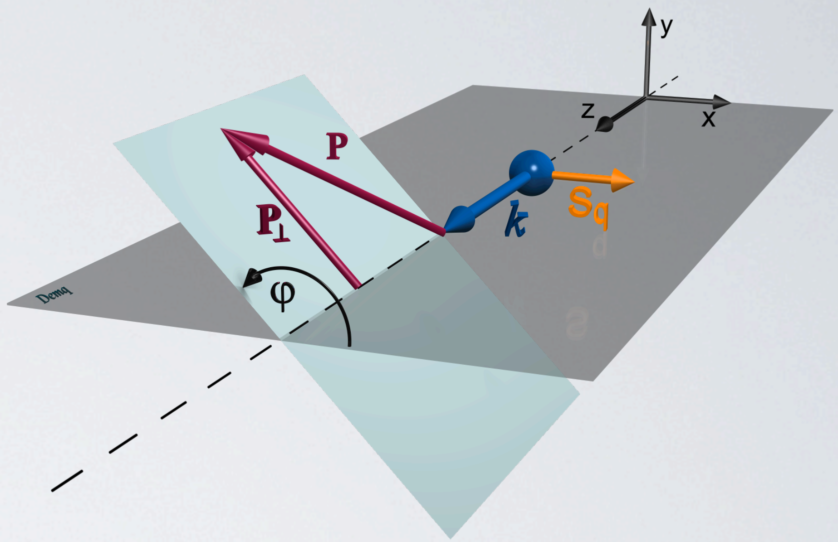
- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.



COLLINS FRAGMENTATION FUNCTION

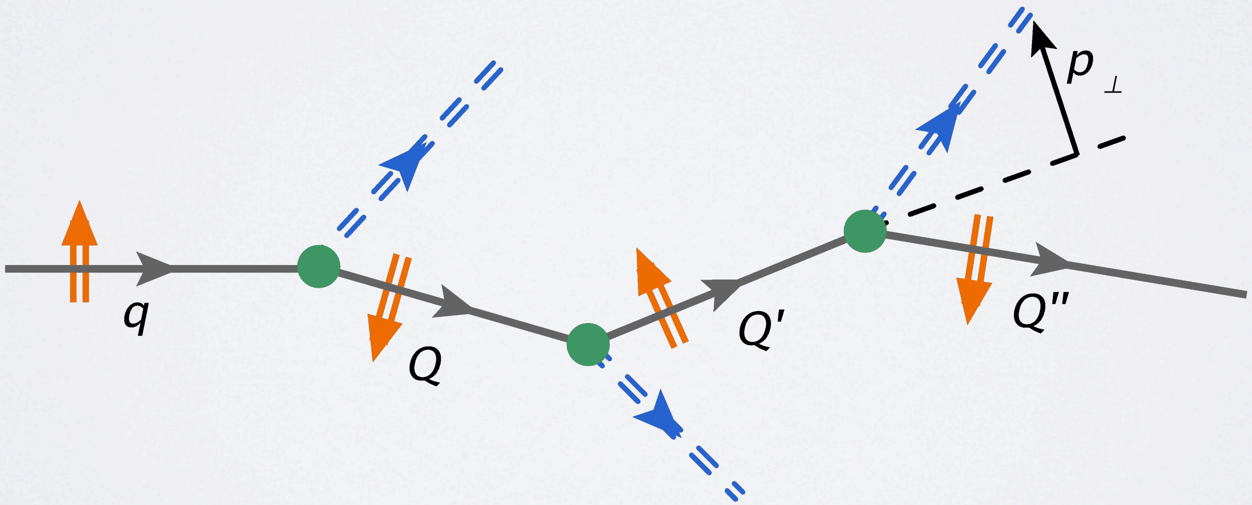
- **Collins Effect:**

  Azimuthal Modulation of the Fragmentation Function of a Transversely Polarized Quark.



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

- **Extend the NJL-jet Model to Include the Quark's Spins.**



- **Model Calculated Elementary Collins Function as Input**

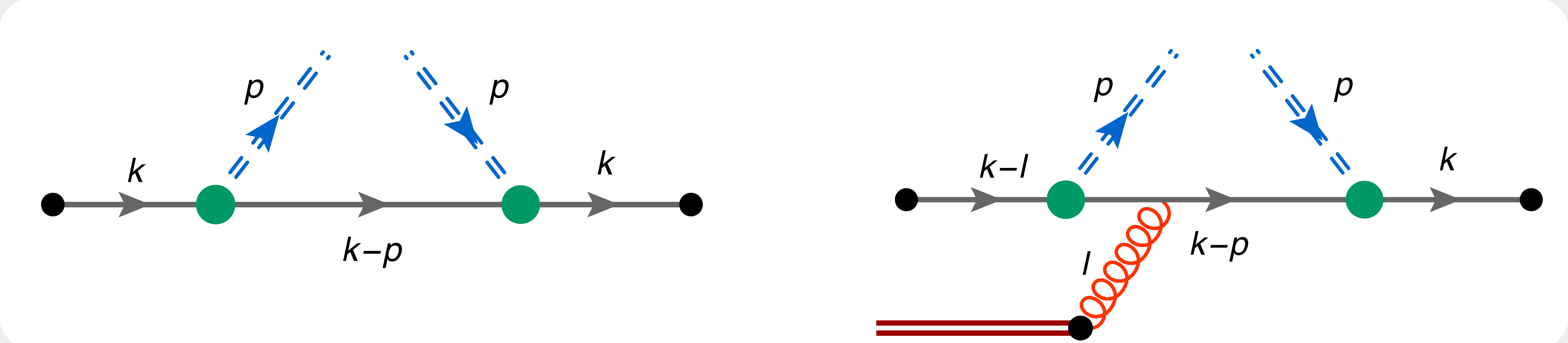
ELEMENTARY POLARIZED SPLITTINGS

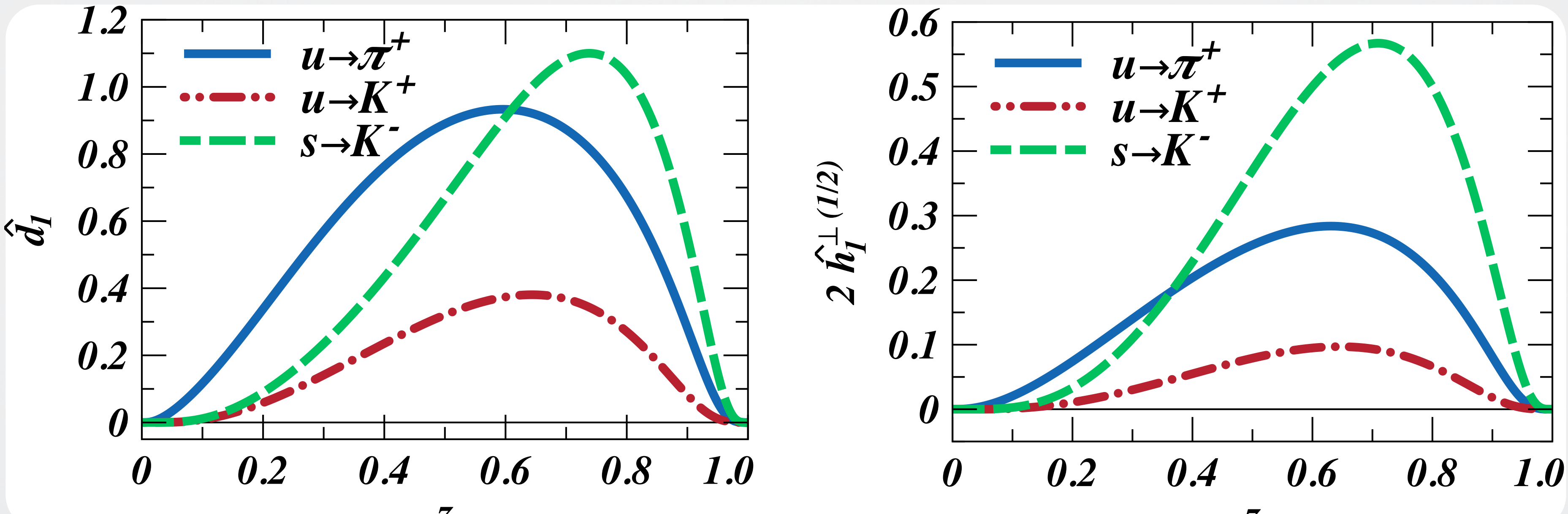
- One-quark truncation of the wavefunction:

Bacchetta et. al., Phys. Lett. B659, 234 (2008).
Gamberg et. al.,Phys.Rev. D68, 051501 (2003).

$$d_1^{h/q}(z,p_\perp^2) = \frac{1}{2}\mathrm{Tr}\left[\Delta_0(z,p_\perp^2)\gamma^+\right]$$

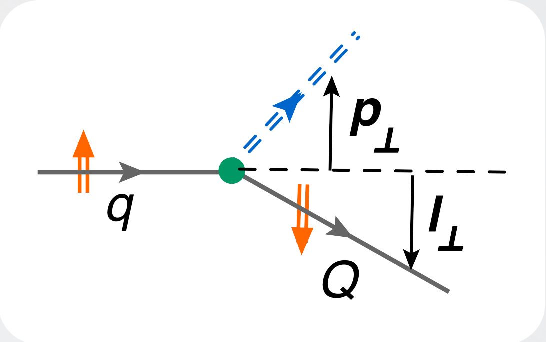
$$\frac{\epsilon_T^{ij} p_{\perp j}}{z m_h}\hat{H}_1^\perp(z,p_\perp^2) = \mathrm{Tr}[\Delta_0(z,\boldsymbol{p}_\perp^2)\imath\sigma^{i-}\gamma_5]$$





QUARK SPIN FLIP PROBABILITY

- Consider Elementary Splitting.
- *Approximation: only tree-level amplitude!*



- Use Lepage-Brodsky Spinors in helicity base to construct the transversely polarized quark spinors:

Y.V. Kovchegov and M. D. Sievert (2012), 1201.5890.

$$U_\chi \equiv \frac{1}{\sqrt{2}}\left[U_{(+z)} + \chi\, U_{(-z)}\right] \qquad \bar{U}_\chi(k,m)U_{\chi'}(k,m) = \delta_{\chi,\chi'} 2m$$

$$(\not{k} - m)\; U_\chi = 0 \qquad W_1\; U_\chi = \chi \frac{m}{2} U_\chi$$

- Where Pauli-Lubanski vector as Lorentz-covariant spin operator:

$$W_\mu \equiv -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} S^{\nu\rho} k^\sigma \qquad S^{\nu\rho} \equiv \frac{\imath}{4}\left[\gamma^\nu, \gamma^\rho\right]$$

- The corresponding matrix elements between *in* and *out* states: $\Psi_{out} = a_1\; U_1(l, M_2) + a_{-1}\; U_{-1}(l, M_2)$

$$\left|\bar{U}_{\chi'}(l, M_2)\gamma^5 U_\chi(k, M_1)\right|^2 = \delta_{\chi,\chi'} \frac{l_x^2}{1-z} + \delta_{\chi,-\chi'} \frac{l_y^2 + (M_2 - (1-z)M_1)^2}{1-z}$$

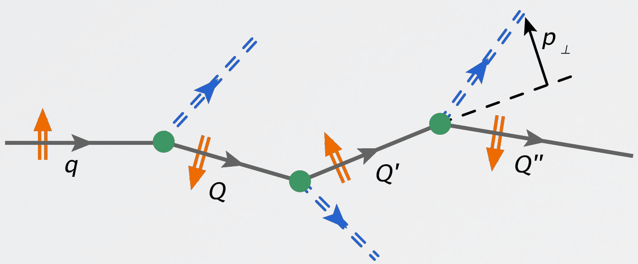
- ***Spin non-flip and flip probabilities are proportional to:***

$$|a_1|^2 \sim l_x^2,\;\; |a_{-1}|^2 \sim l_y^2 + (M_2 - (1-z)M_1)^2$$

MC SIMULATIONS

- MC Simulations to Calculate the Average Number Density

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$



$$\equiv \frac{\sum_{N_{Sims}} N_{q^\uparrow}^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2; \varphi, \varphi + \Delta\varphi)}{N_{Sims}}$$

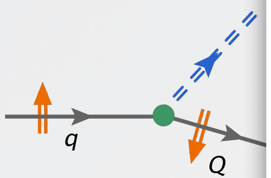
- Simulation Parameters:
  - Discretize $z \in [0,1], P_\perp^2 \in [0,1]$ and $\varphi \in [0, 2\pi)$ with $N_{Bins} = 100$.
  - Number of emitted hadrons in each decay chain: $N_{Links} = \{1, 2, 6\}$.
  - Toy model simulation: $\{u, d\} \to \{\pi\}$ and number of decay chains: $N_{Sims} = 10^{11}$.
  - Full model simulation: $\{u, d, s\} \to \{\pi, K\}$ and number of decay chains: $N_{Sims} = 10^{12}$.

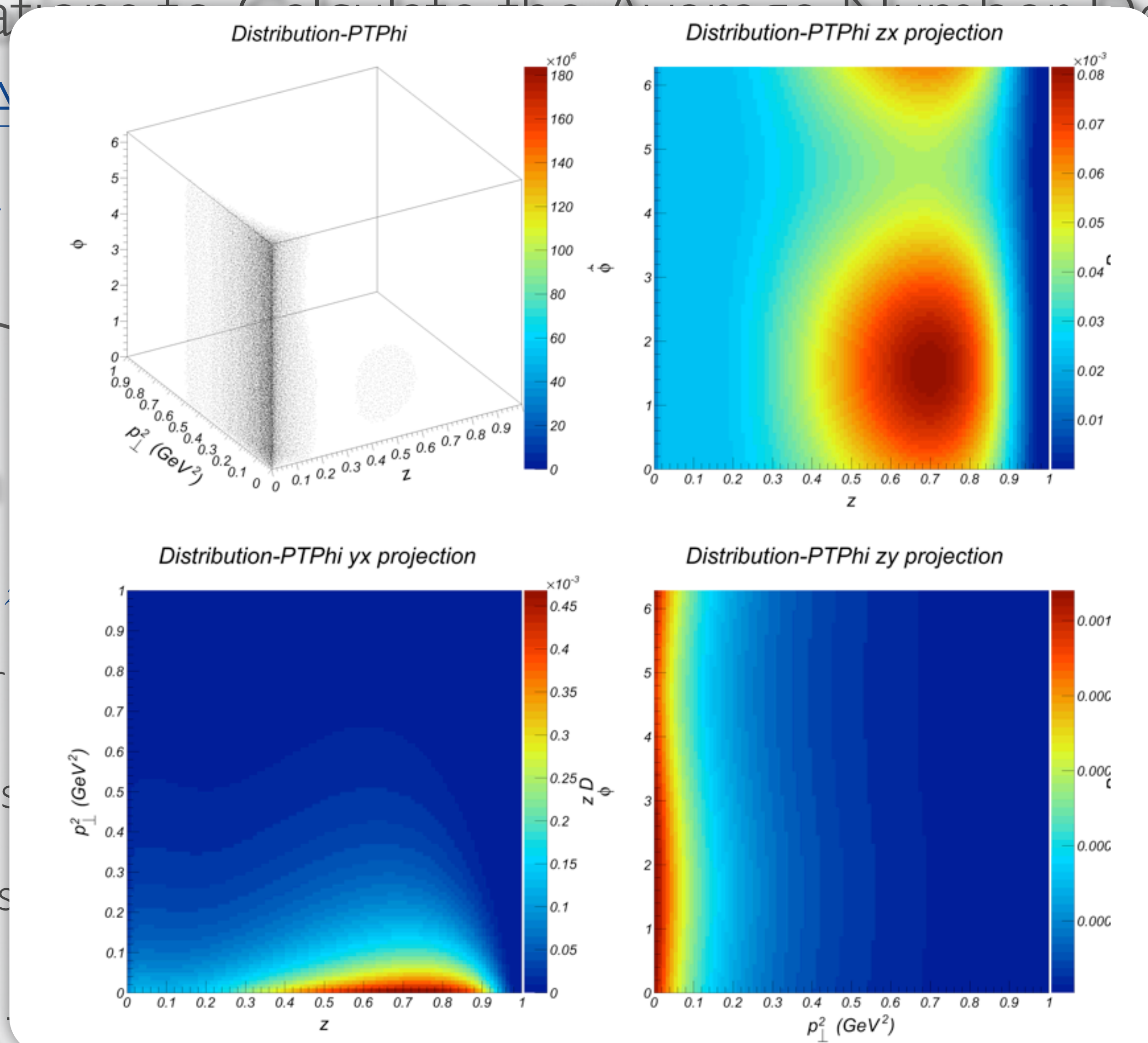
- Simulation Time for the Full Model ~ 10 days on 50 cores of Intel Core i7-920 CPUs.

MC SIMULATIONS

- MC Simulati… ensity

$D_{h/q^{\uparrow}}(z, P_\perp^2, \varphi)\,\Delta z\,\frac{\Delta}{}$ … $+ \Delta P_\perp^2; \varphi, \varphi + \Delta\varphi)$



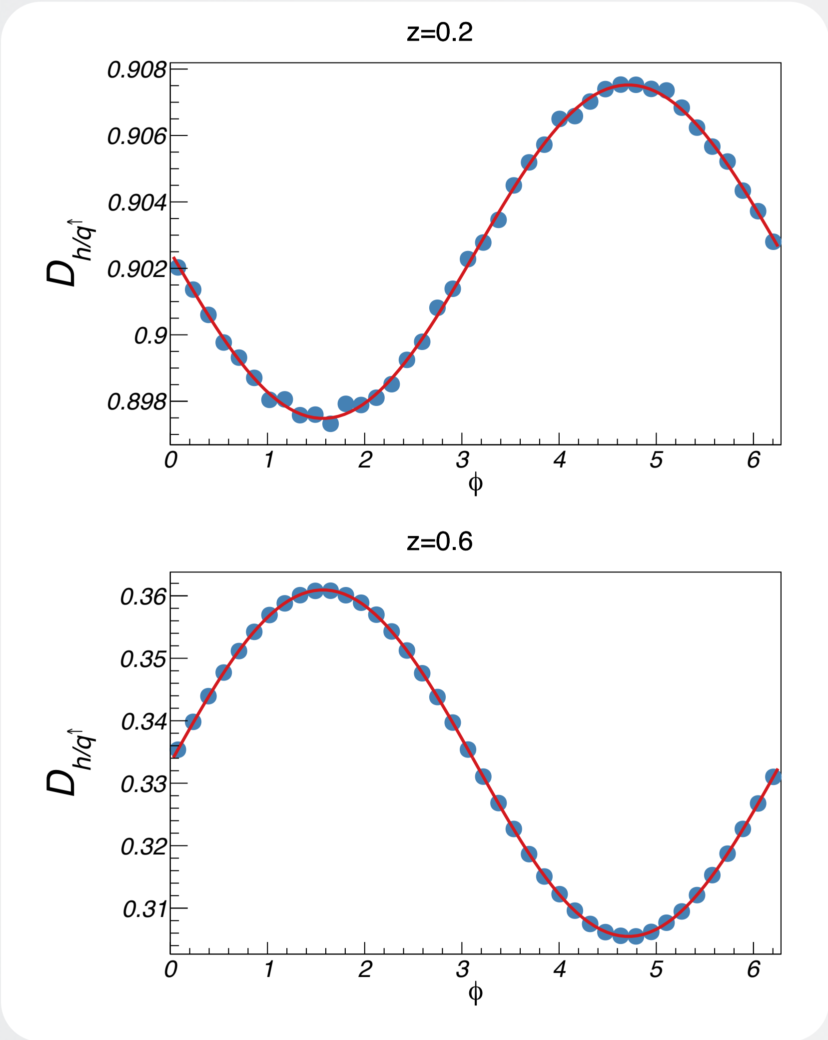


- Simulation
  - Discretize
  - Number of
  - Toy model s … $ms = 10^{11}$.
  - Full model s … $ms = 10^{12}$.
- Simulation Time for the Full Model – 10 days on 30 cores of Intel Core i7-920 CPUs.

INTEGRATED POLARIZED FRAGMENTATIONS

- First: Integrate Polarized Fragmentations over $P_\perp^2$

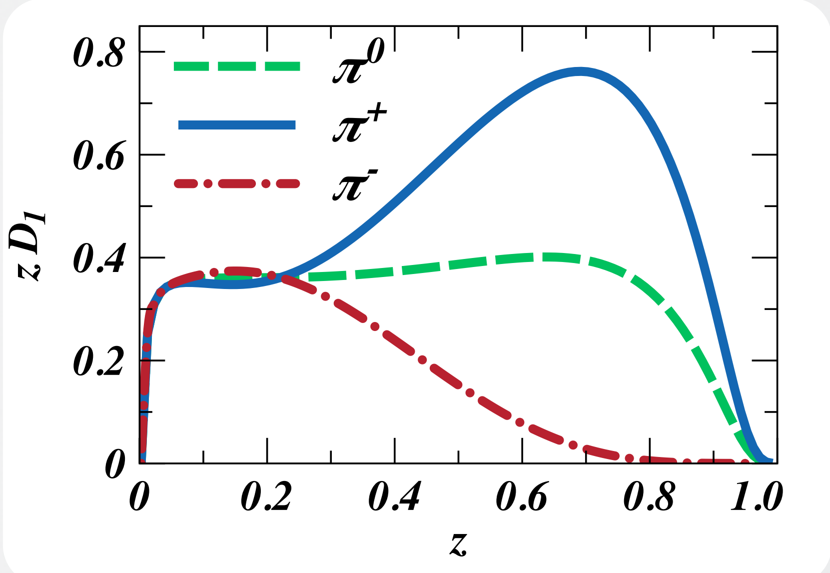
$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2 \; D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z) \; - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$



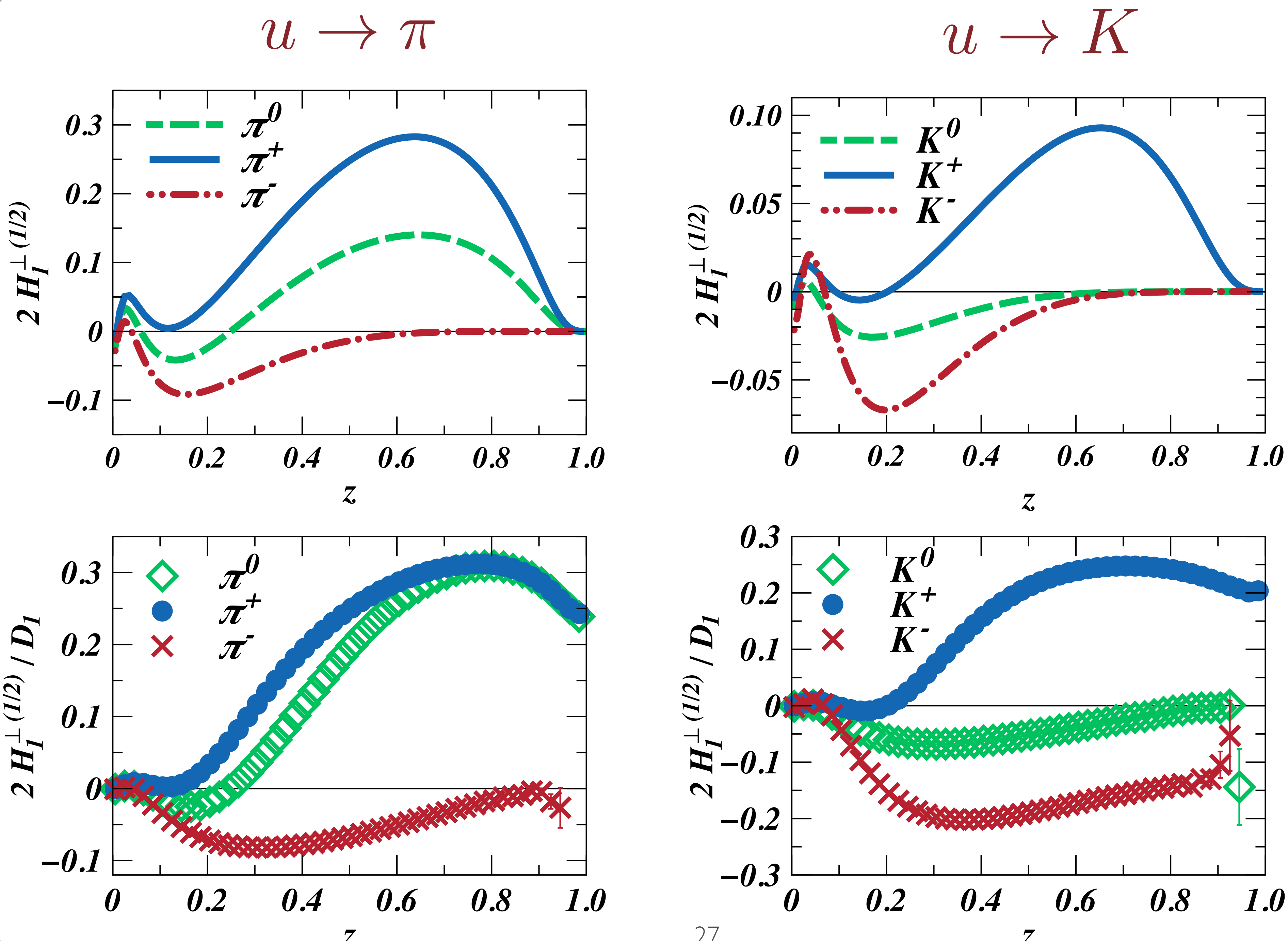
$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2 \; D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $\boxed{c_0 + c_1 \sin(\varphi)}$



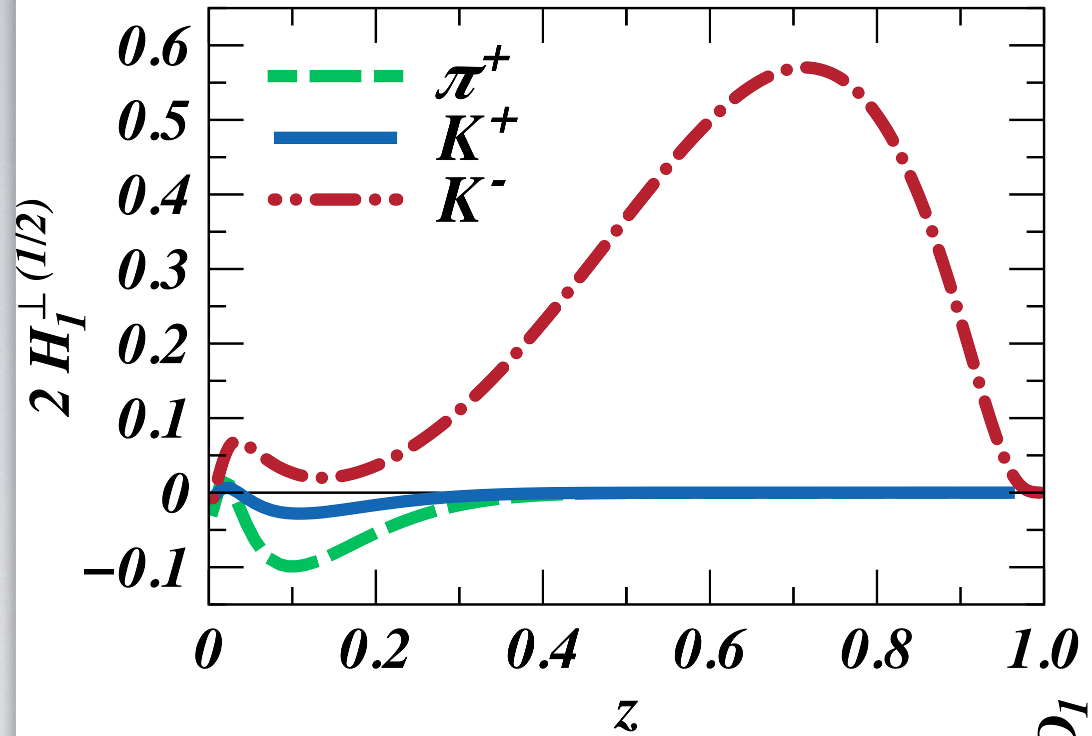
1/2 MOMENT OF COLLINS FUNC.



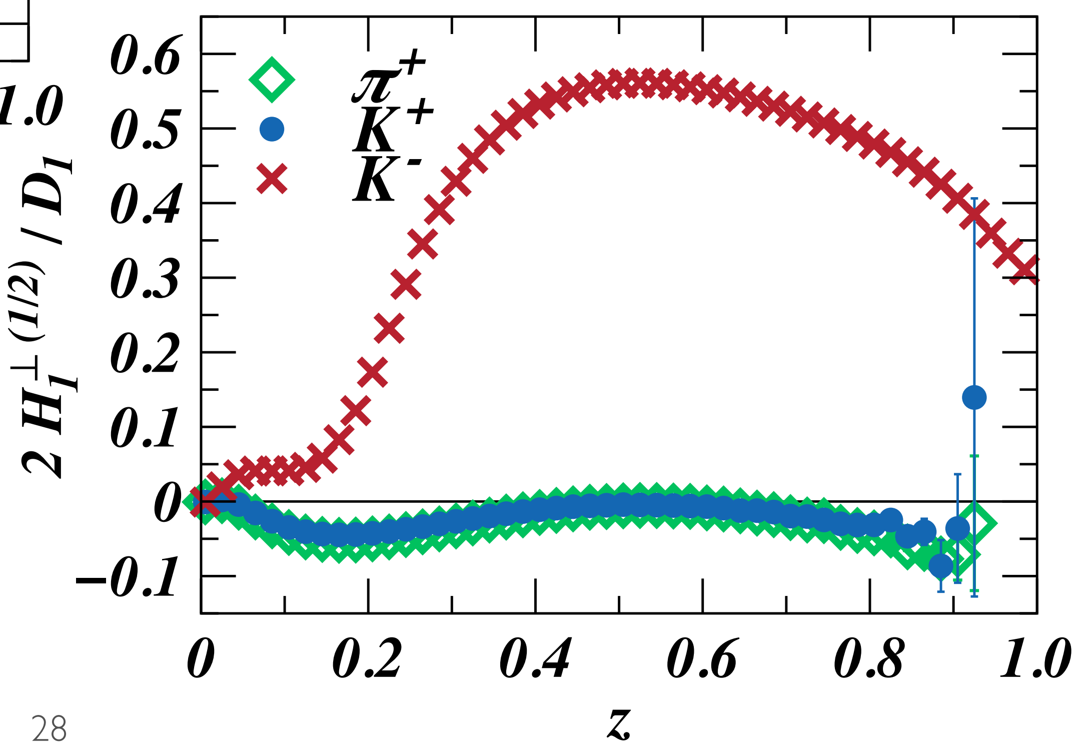
- **Collins**
- **Ratio**

1/2 MOMENT OF COLLINS FUNC.

- **Collins** $s \to \pi, K$



- **Ratio**

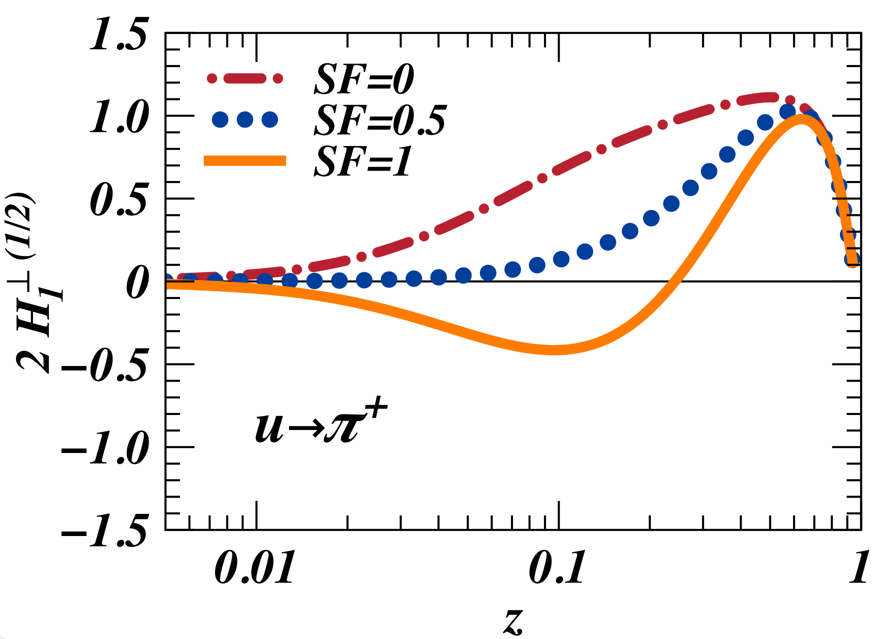


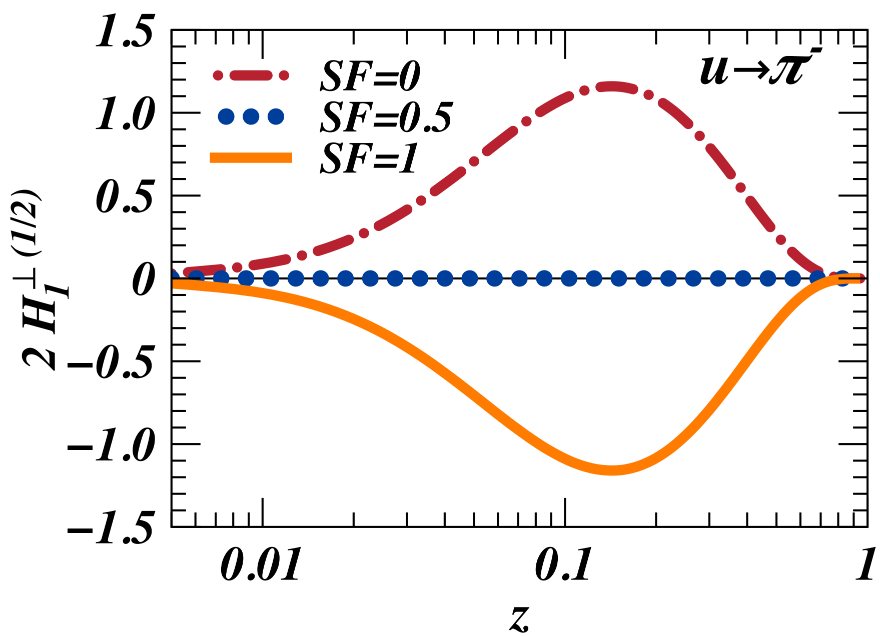
ROLE OF THE QUARK SPIN FLIP

- EMPLOY A TOY MODEL TO STUDY THE SPIN FLIP EFFECTS

$$d^{(toy)}_{h/q^{\uparrow}}(z, p_{\perp}^2) = d_1^{h/q}(z, p_{\perp}^2)(1 + 0.9 \sin\varphi)$$

- Set the Quark Spin Flip Probability as a Constant: SF
- Simulations with only light quarks to pions: $\{u, d\} \to \pi^{\pm,0}$



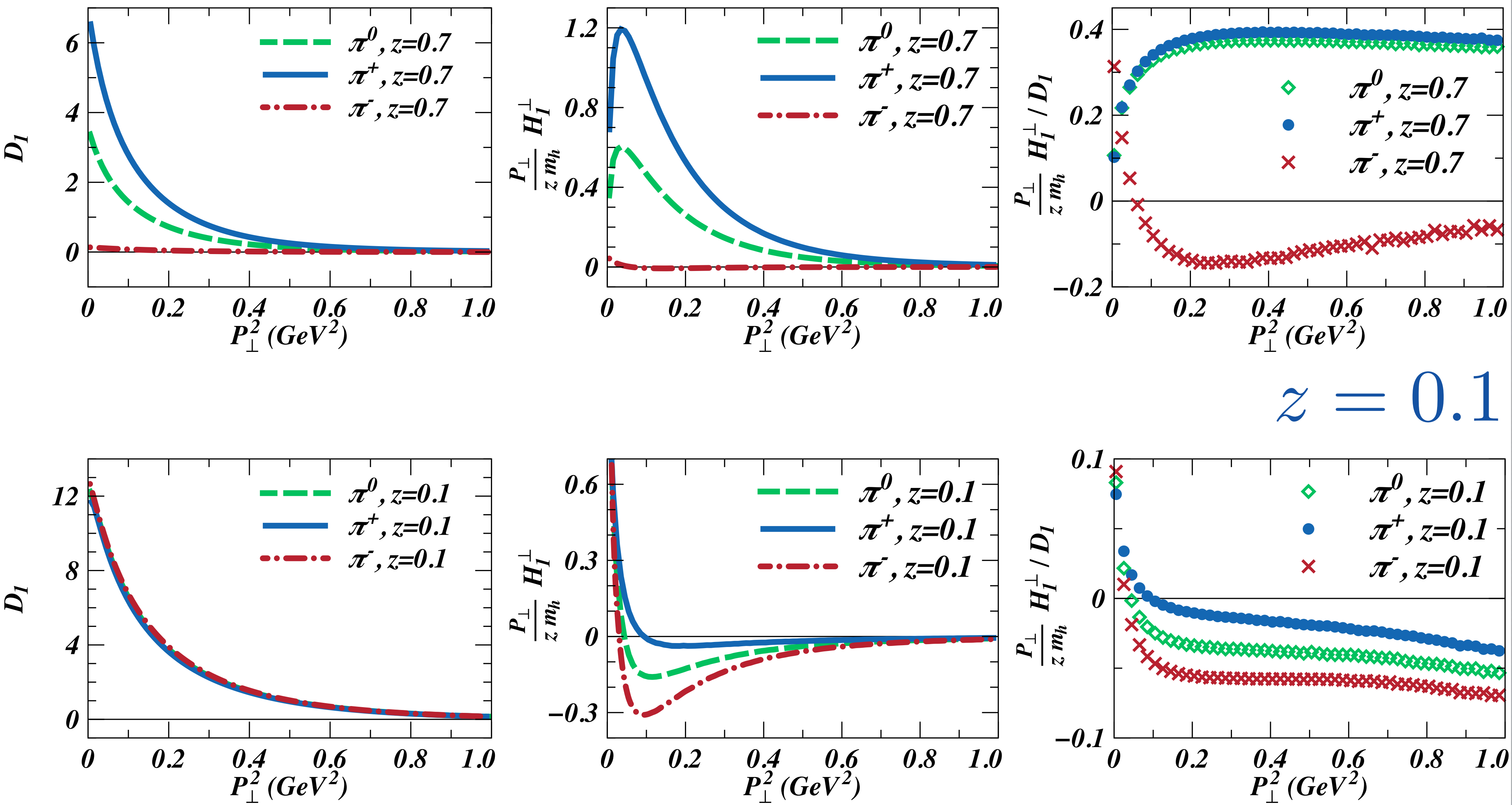


- Preferential Quark Spin Flip is ESSENTIAL to Generate Opposite Signed Collins 1/2 Moments!

TMD FRAGMENTATION FUNC. FOR PION

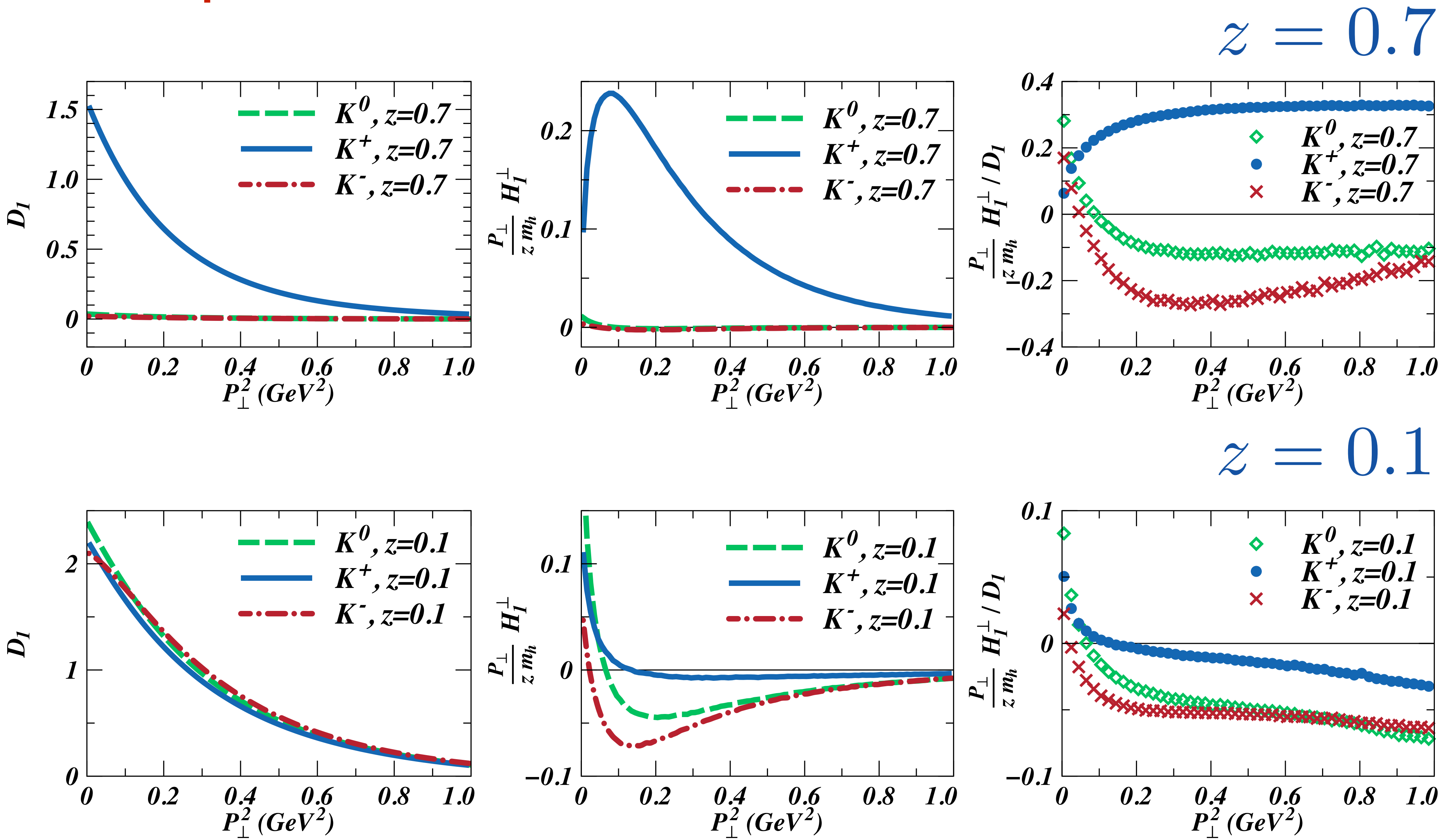
- Unpolarized
- Collins
- Ratio

$z = 0.7$



TMD FRAGMENTATION FUNC. FOR KAONS

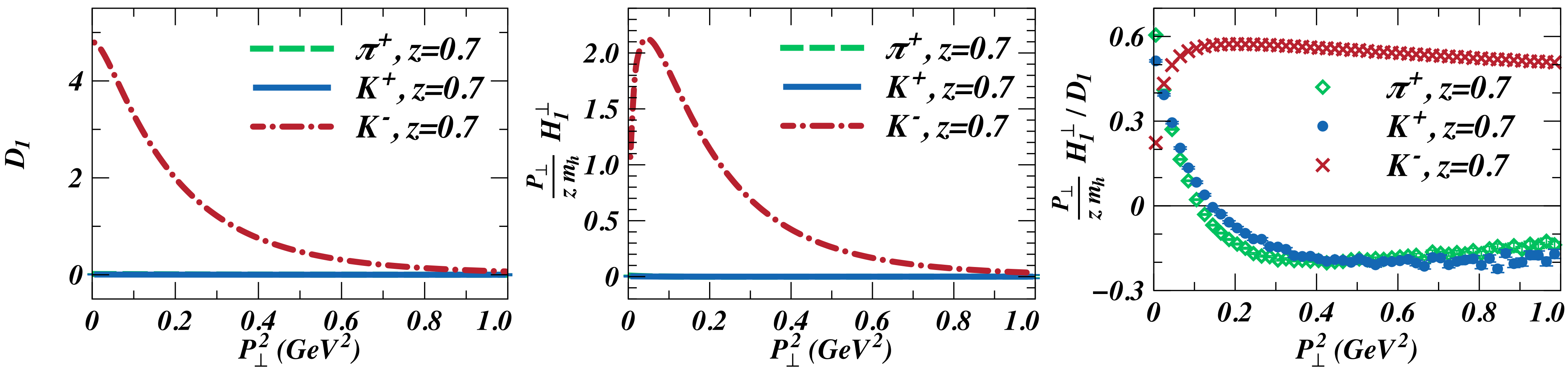
- Unpolarized
- Collins
- Ratio



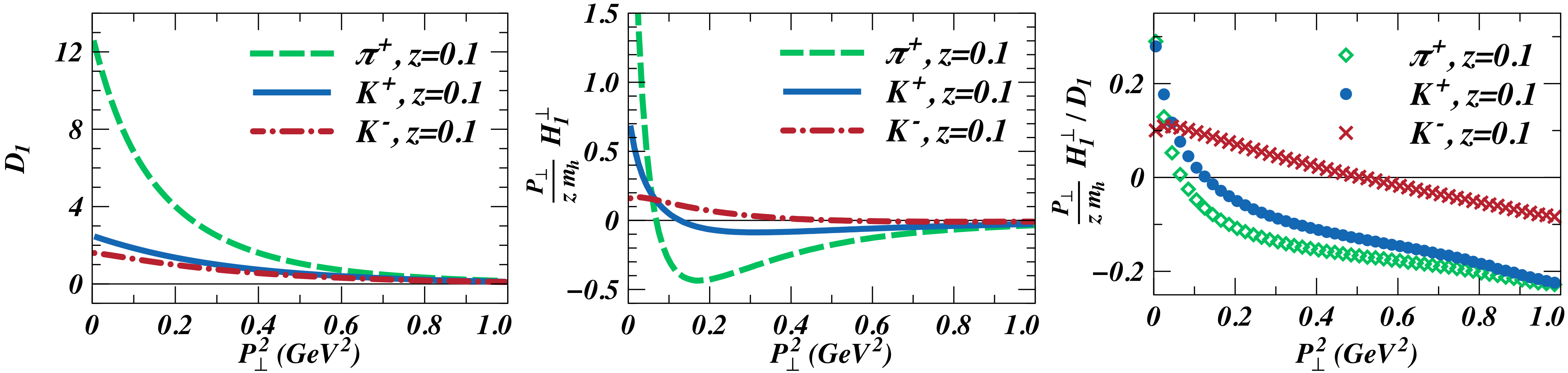
TMD FRAGMENTATION FUNC. FOR S

- Unpolarized
- Collins
- Ratio

$z = 0.7$



$z = 0.1$



GLOBAL FITS TO EXPERIMENTAL DATA

Anselmino et al., Nuclear Physics B (Proc. Suppl.) 191 (2009) 98–107.

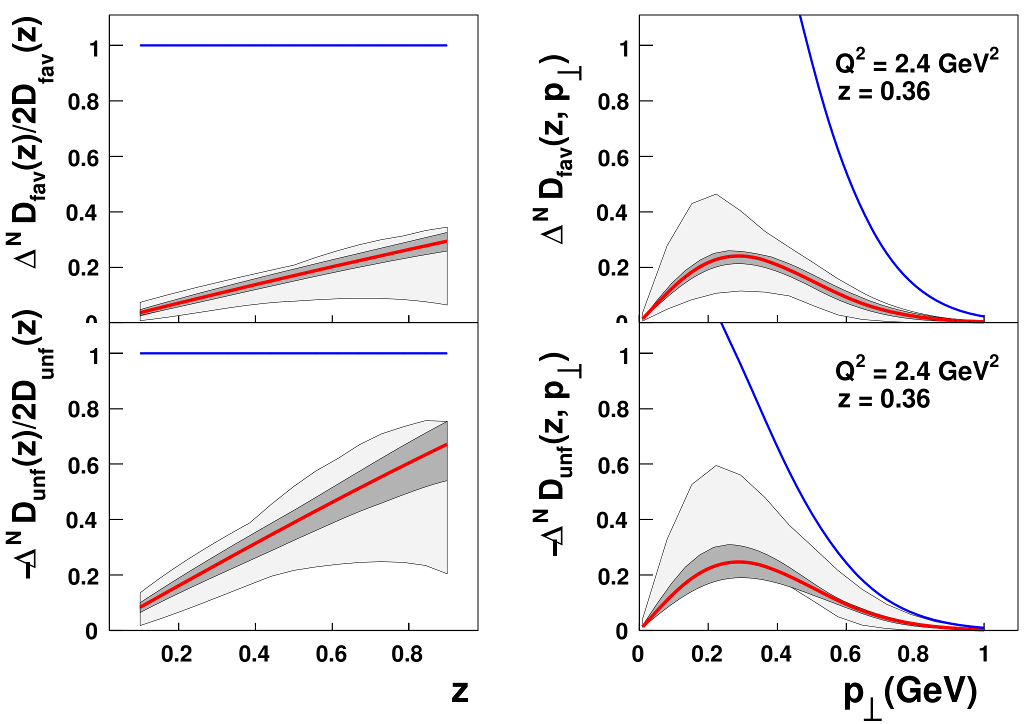
Consider e^+e^- and SIDIS

BELLE, R. Seidl et al., Phys. Rev. D78 (2008) 032011.
HERMES, M. Diefenthaler, Proc. of DIS2007 (2007).
COMPASS, M. Alekseev et al., arXiv:0802.2160.

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{z m_h} H_1^{\perp h/q}(z,p_\perp) \qquad \Delta^N D_{h/q^\uparrow}(z) = \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z,p_\perp) = 4H_1^{\perp(1/2)}(z)$$

$$\Delta^N D_{h/q^\uparrow}(z)/2D_1(z) = 2H_1^{\perp(1/2)}(z)/D_1(z)$$

Parametrizations and the fits.



Using Gaussian Ansatz:

$$D_1^{h/q}(z,p_\perp) \sim \frac{e^{-p_\perp^2/<p_\perp^2>}}{\pi < p_\perp^2 >}$$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) \sim \frac{p_\perp}{M} e^{-p_\perp^2/M^2} \frac{e^{-p_\perp^2/<p_\perp^2>}}{\pi < p_\perp^2 >}$$

$$D_{\pi^+/u,\bar{d}} = D_{\pi^-/d,\bar{u}} = D_{fav}$$

$$D_{\pi^+/d,\bar{u}} = D_{\pi^-/u,\bar{d}} = D_{\pi^\pm/s,\bar{s}} = D_{unf}$$

THE SCHÄFER-TERYAEV SUM RULE

- Naïve Schäfer-Teryaev Sum Rule: A. Schafer and O. Teryaev, Phys.Rev. D61, 077903 (2000)

  Transverse Momentum Conservation of Produced Hadrons.

$$ST_q \equiv \sum_h \int_0^1 dz\ H_{1,(h/q)}^{\perp(1)}(z) = 0 \qquad H_{1,(h/q)}^{\perp(1)}(z) \equiv \pi \int_0^\infty dP_\perp^2\ \frac{P_\perp^2}{2zm_h} H_1^{\perp h/q}(z, P_\perp^2)$$

- In Our Results the Sum Rule is NOT Satisfied:

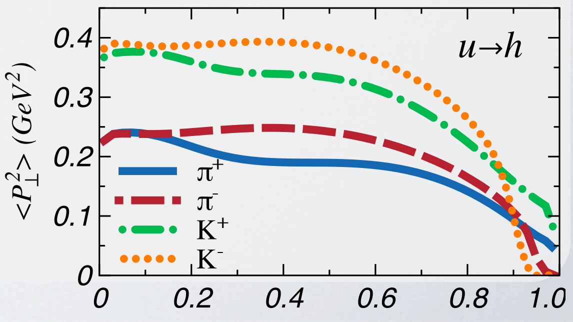
  Transverse Momentum Conservation is Explicitly Enforced.

$$\boxed{ST_u = 0.07 \qquad ST_s = 0.21}$$

- Need Quark to Quark Contribns.: S. Meissner, A. Metz, and D. Pitonyak, Phys.Lett. B690, 296 (2010)

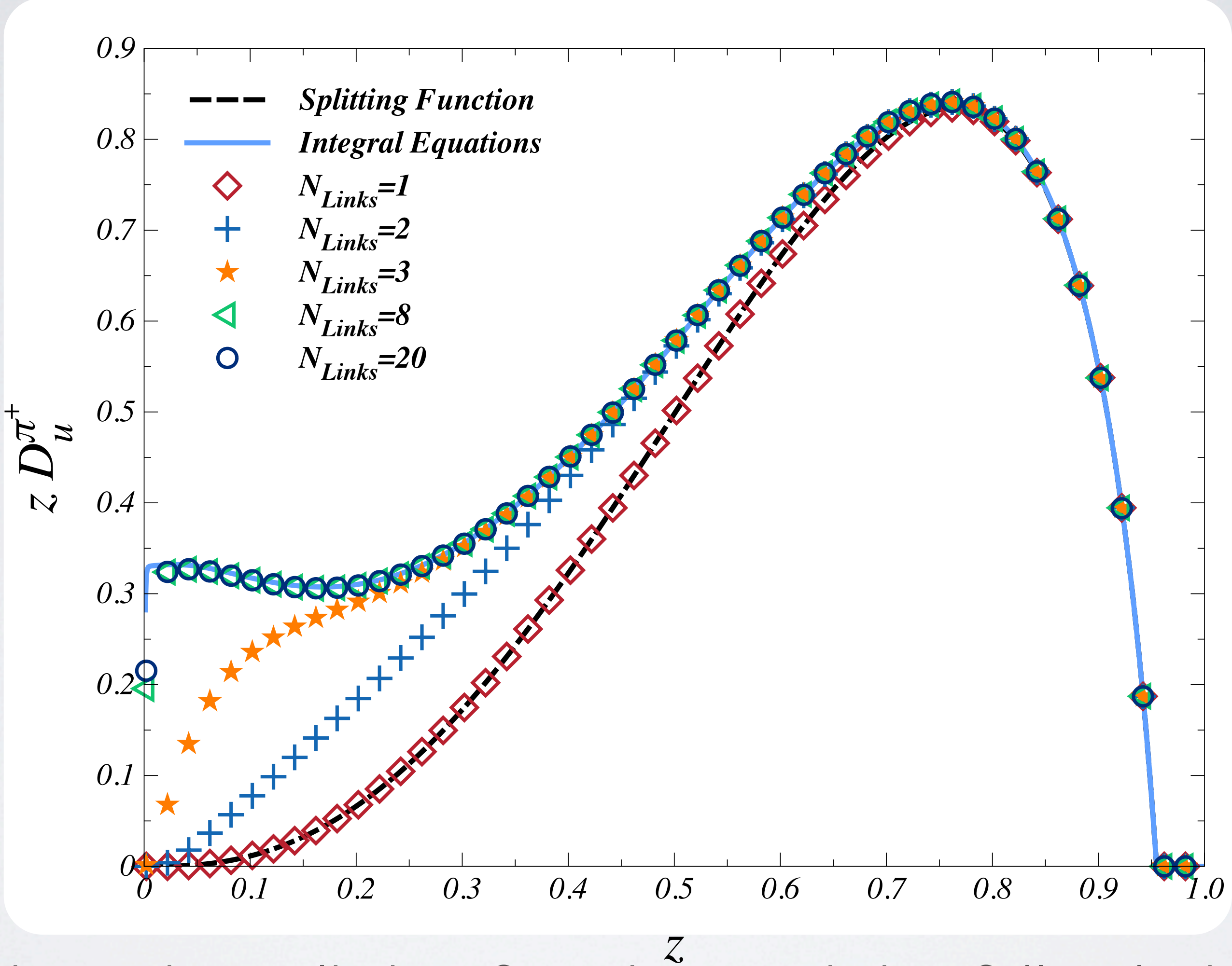
  Include the Transverse Momentum of the Remnant Quark.

$$\sum_h \int dz\ H_{1,(h/q)}^{\perp(1)}(z) + \sum_Q \int dz\ 2H_{1,(Q/q)}^{\perp(1)}(z) = 0$$



DEPENDENCE ON CHAIN CUTOFF

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.

- The low z region is saturated with just a few emissions.

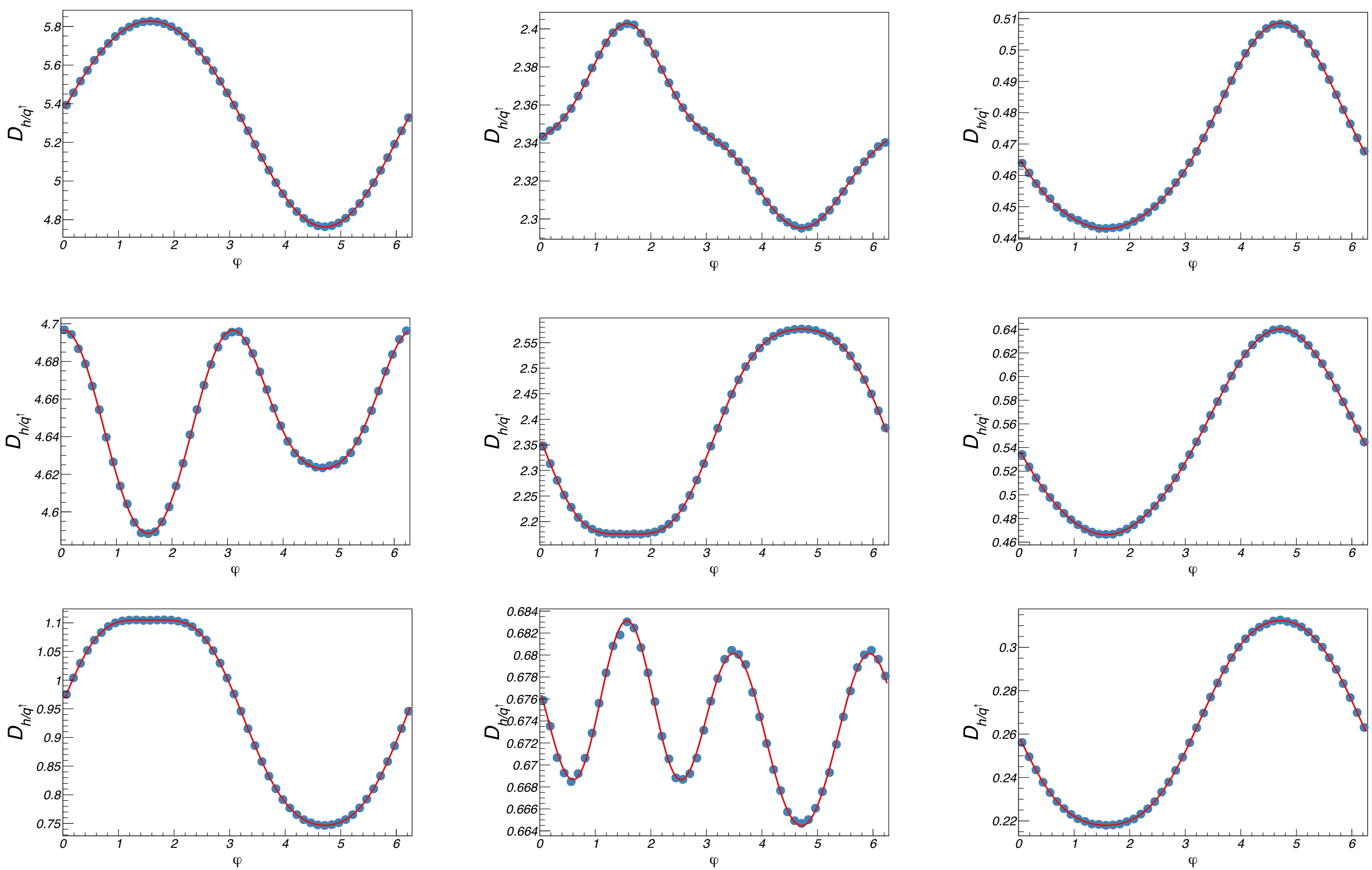
HIGHER ORDER COLLINS MODULATIONS IN TRANSVERSELY POLARIZED QUARK FRAGMENTATION

HIGHER ORDER COLLINS MODULATIONS

- TMD $D_{h/q^\uparrow}(z, P_\perp^2, \varphi)$ Exhibit Higher Order Modulations.

$$N_{Links} = 6 \qquad z = 0.2$$

$$P_\perp^2 = 0.04\text{GeV}^2 \qquad P_\perp^2 = 0.16\text{GeV}^2 \qquad P_\perp^2 = 0.49\text{GeV}^2$$



$u \to \pi^+$

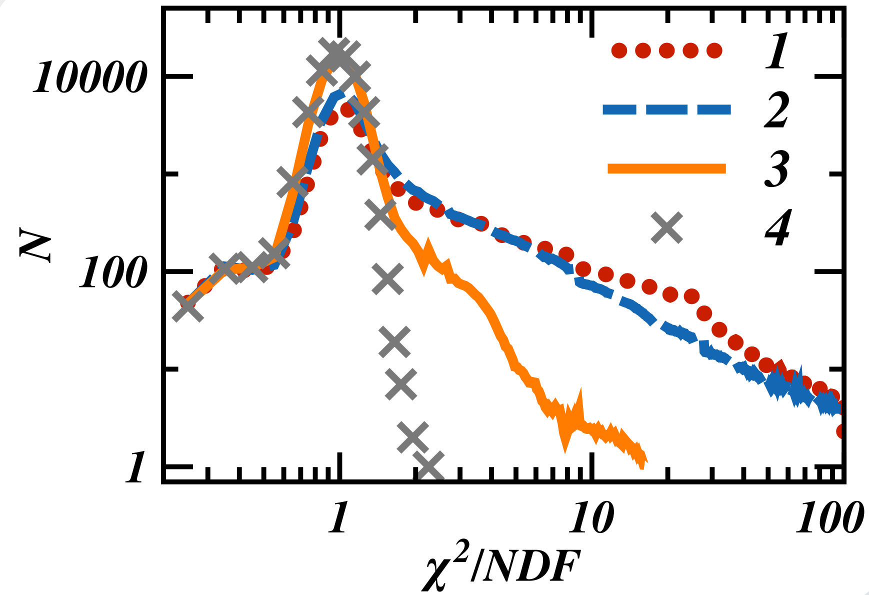
$u \to \pi^-$

$s \to K^+$

FITS TO HIGHER ORDER COLLINS MODULATIONS

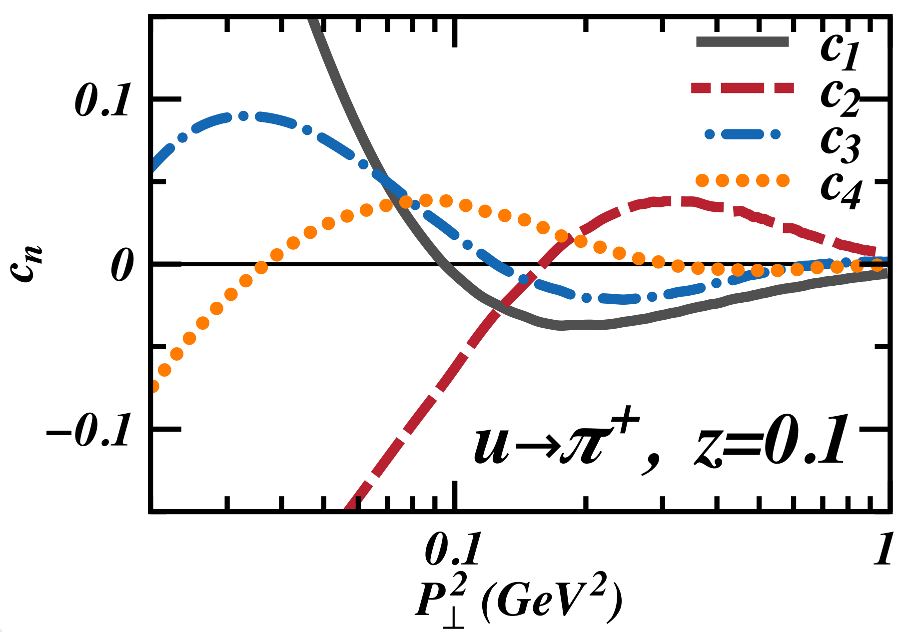
- Test Higher Order Polynomial in $\sin\varphi$: $D_{h/q^{\uparrow}} = \sum_{0}^{L} c_n \sin^n \varphi$

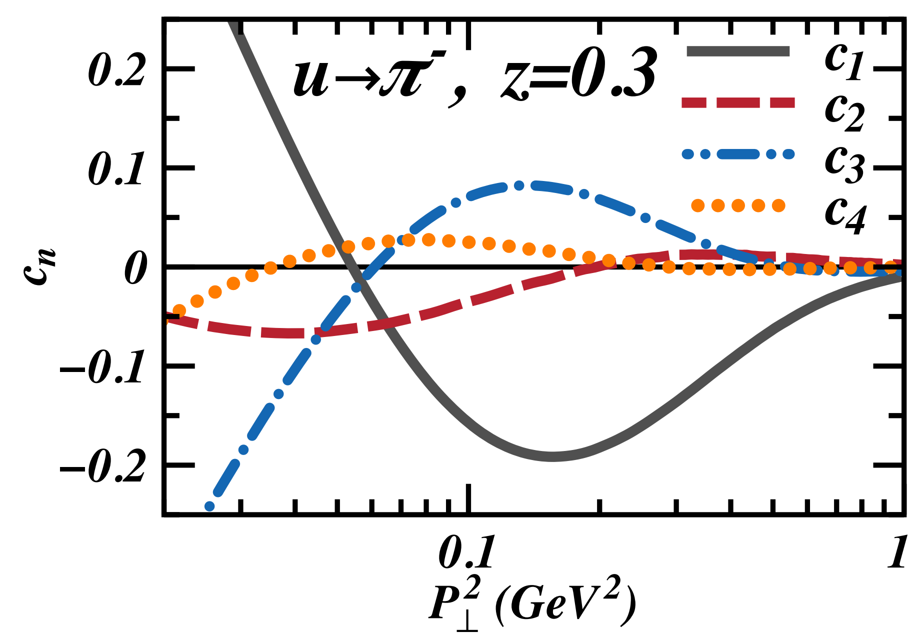
- Histogram of χ^2/NDF extracted from fits to ALL slices of z and $P_\perp^2$ using polynomial forms of various orders.



- Total: 7×10⁴ fits for $u \rightarrow h$!

- Extracted Coefficients from Fits:





WHAT IS THE ***SOURCE*** OF THE MODULATIONS ?

- EMPLOY A TOY MODEL TO STUDY THE SPIN FLIP EFFECTS
- Use a very Large Elementary Collins Function

$$d^{(toy)}_{h/q^{\uparrow}}(z, p_{\perp}^2) = d_1^{h/q}(z, p_{\perp}^2)(1 + 0.9 \sin\varphi)$$

- Set the quark spin flip constant to maximum:

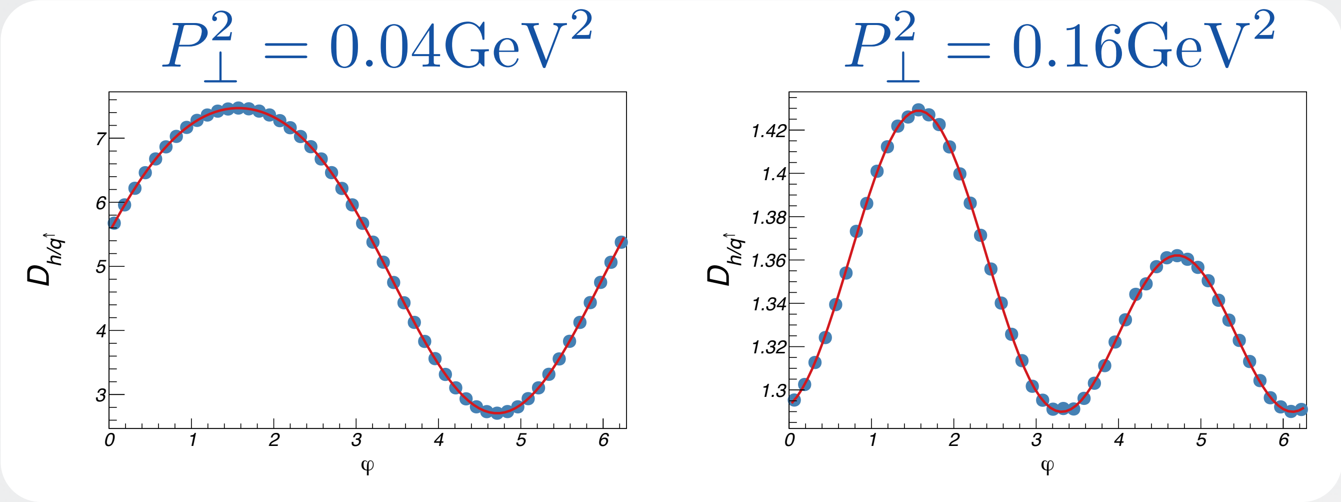
$$SF = 1$$

- Simulations with only light quarks to pions:

$$\{u, d\} \to \pi^{\pm,0}$$

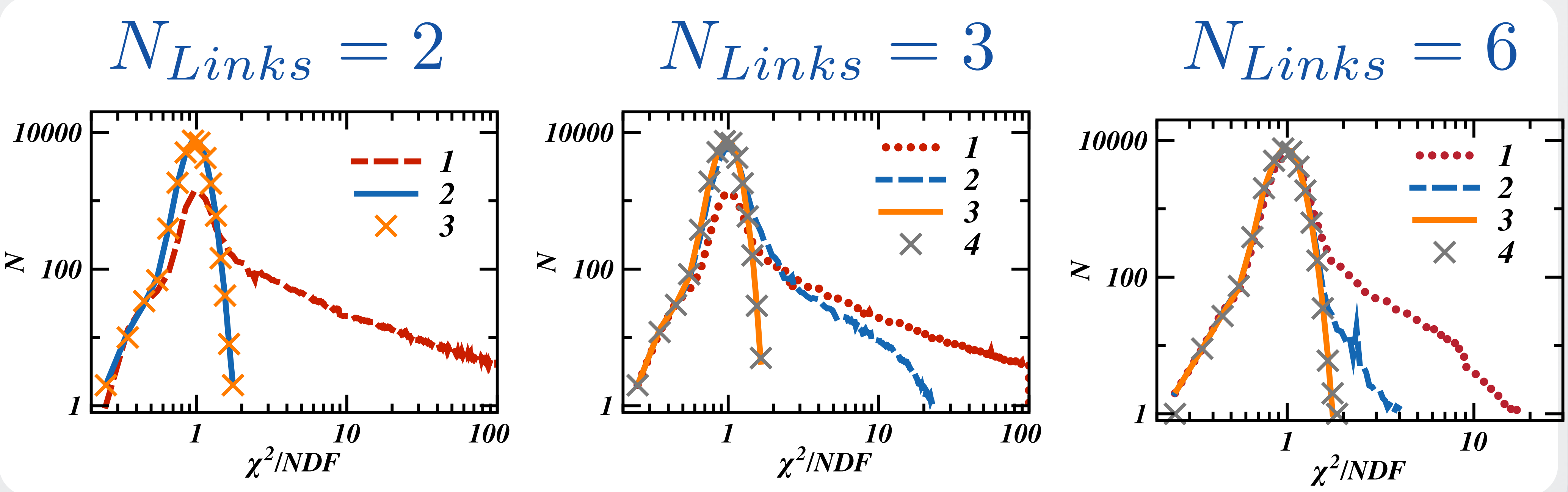
WHAT IS THE SOURCE OF THE MODULATIONS ?

- Sample for $u \to \pi^+$ and $N_{Links} = 2$



$$z = 0.3$$

- Employ the histograms for χ^2/NDF from fits:



MODULATION SOURCE I: ***QUARK RECOIL***

- For the <u>TOY MODEL</u>, with each additional hadron emission the modulation increases by one order.

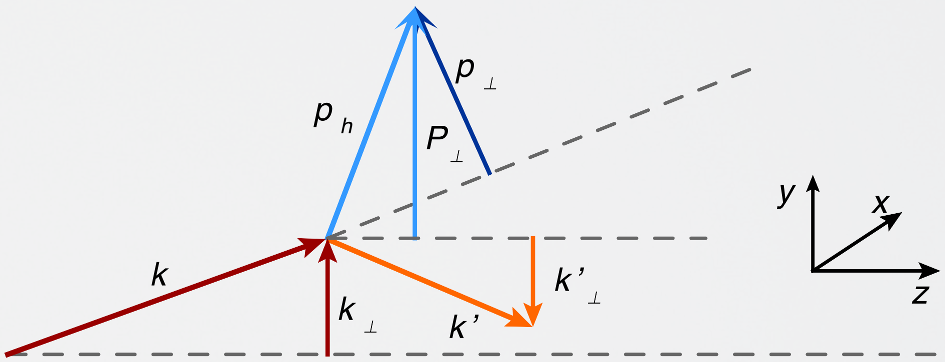
- At each hadron emission:

$$d_{Q/q^\uparrow}(1-z, -\boldsymbol{p}_\perp) = d_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

- This can be attributed to the modulation of the remnant quark through recoil transverse momentum:

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



- Thus the hadrons emitted in n-th step acquire $\sin^n \varphi$ modulation

MODULATION SOURCE II: ***QUARK SPIN FLIP***

- The Full Model Exhibits *4-th* Order Modulations with $N_{Links} = 2$
- The Remaining 2 Orders in Full Model attribute to SF
- Easy to See From:

$$SF \sim l_y^2 + (M_2 - (1-z)M_1)^2$$

$$l_y = -p_y = -p_\perp \sin\varphi$$

- In the Full Model with ***R*** hadrons emissions:

$$\boxed{D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = \sum_{n=0}^{1+3(R-1)} c_n \sin^n \varphi}$$

CONCLUSIONS

- We extended NJL-jet Model to describe the Collins effect using our Monte Carlo Framework.
- We presented the ***first model calculations*** of ***Unfavored*** and Favored Collins FFs with NO fitted parameters to fragmentation data!
- The Resulting 1/2 Moments of Unfavored Collins Functions of opposite sign to that of Favored Collins Function.
- Remnant quark spin flip - a KEY to generating Unfavored Collins FF.
- Naïve Schäfer-Teryaev Sum Rule does NOT hold in our model: the final remnant quark posses significant transverse momentum.
- We Found *Higher Order Collins Modulations* in TMD Polarized FFs: a direct consequence of multi-hadron emission in quark-jet hadronization framework.
- QCD evolution of Collins function is ***not*** known yet!

OUTLOOK

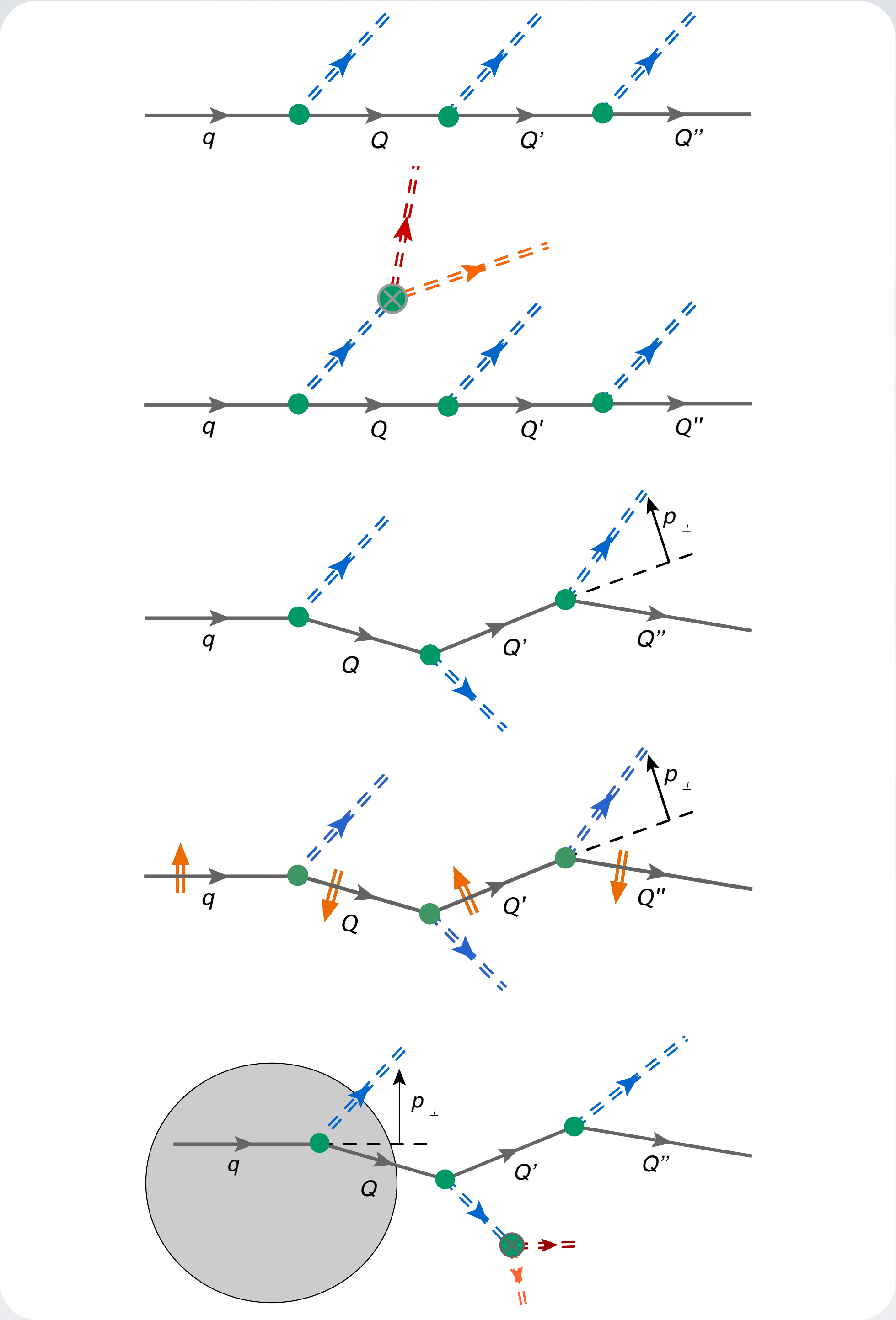
☑ 2009

☑ 2010

☑ 2011

☑ 2012

☐ 201x



Ito et al. Phys.Rev.D80:074008,2009

Matevosyan et al.
Phys.Rev.D83:074003, 2011

Matevosyan et al.
Phys.Rev.D83:114010, 2011

Matevosyan et al.
Phys.Rev. D85:014021, 2012

Matevosyan et al.
arXiv:1205.5813, 2012

Thanks!

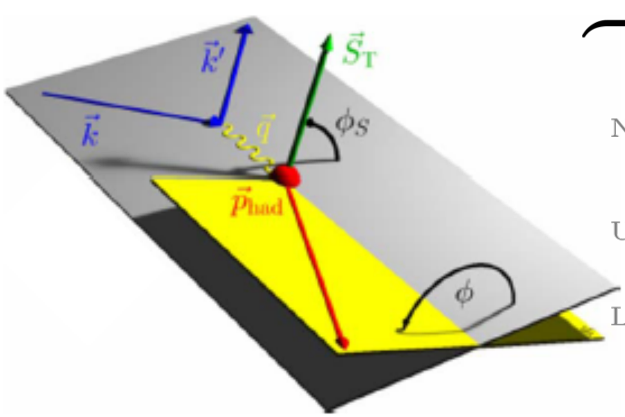
BACK-UP SLIDES

SIDIS Cross Section (lower twists)

Bacchetta et al. arXiv:hep-ph/0611265

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} \Big/ \left[\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right] = \\
F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
+\quad S_L \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
+\quad S_L\lambda_e \left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
+\quad S_T \Big[\sin(\phi_h-\phi_S)\left(\boxed{F_{UT,T}^{\sin(\phi_h-\phi_S)}} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,\boxed{F_{UT}^{\sin(\phi_h+\phi_S)}} + \varepsilon\,\sin(3\phi_h-\phi_S)\,\boxed{F_{UT}^{\sin(3\phi_h-\phi_S)}} \\
+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
+\quad S_T\lambda_e \left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,\boxed{F_{LT}^{\cos(\phi_h-\phi_S)}} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]
\end{aligned}
$$

18 Structure Functions!



PDF/TMD(x, p_T)

N	U (q)	L (q)	T (q)
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$\boxed{f_{1T}^\perp}$	$\boxed{g_{1T}}$	$\boxed{h_1}$, $\boxed{h_{1T}^\perp}$

$\otimes$

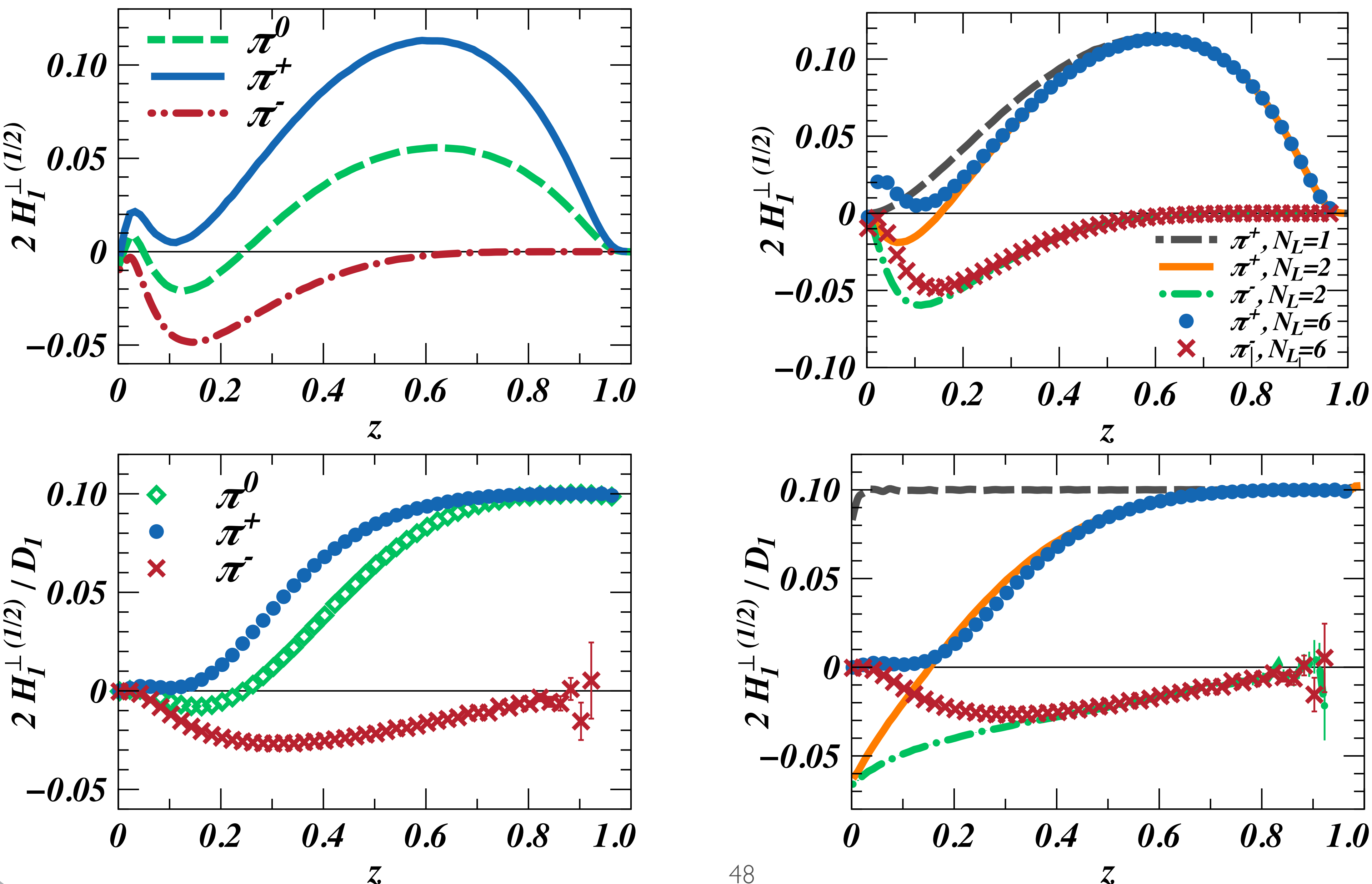
FF(z, K_T)

U (h)	L (h)	T (h)	q'
$\boxed{D_1}$		$D_{1T}^\perp$	U
	G_1	G_{1T}	L
$\boxed{H_1^\perp}$	$H_{1L}^\perp$	$H_1, H_{1T}^\perp$	T

TOY MODEL RESULTS

$$d^{(toy)}_{h/q^{\uparrow}}(z,p_{\perp}^2) = d_1^{h/q}(z,p_{\perp}^2)(1+0.1\sin\varphi)$$

$N_{Links} = 6$ $\qquad$ N_{Links} *Dependence*

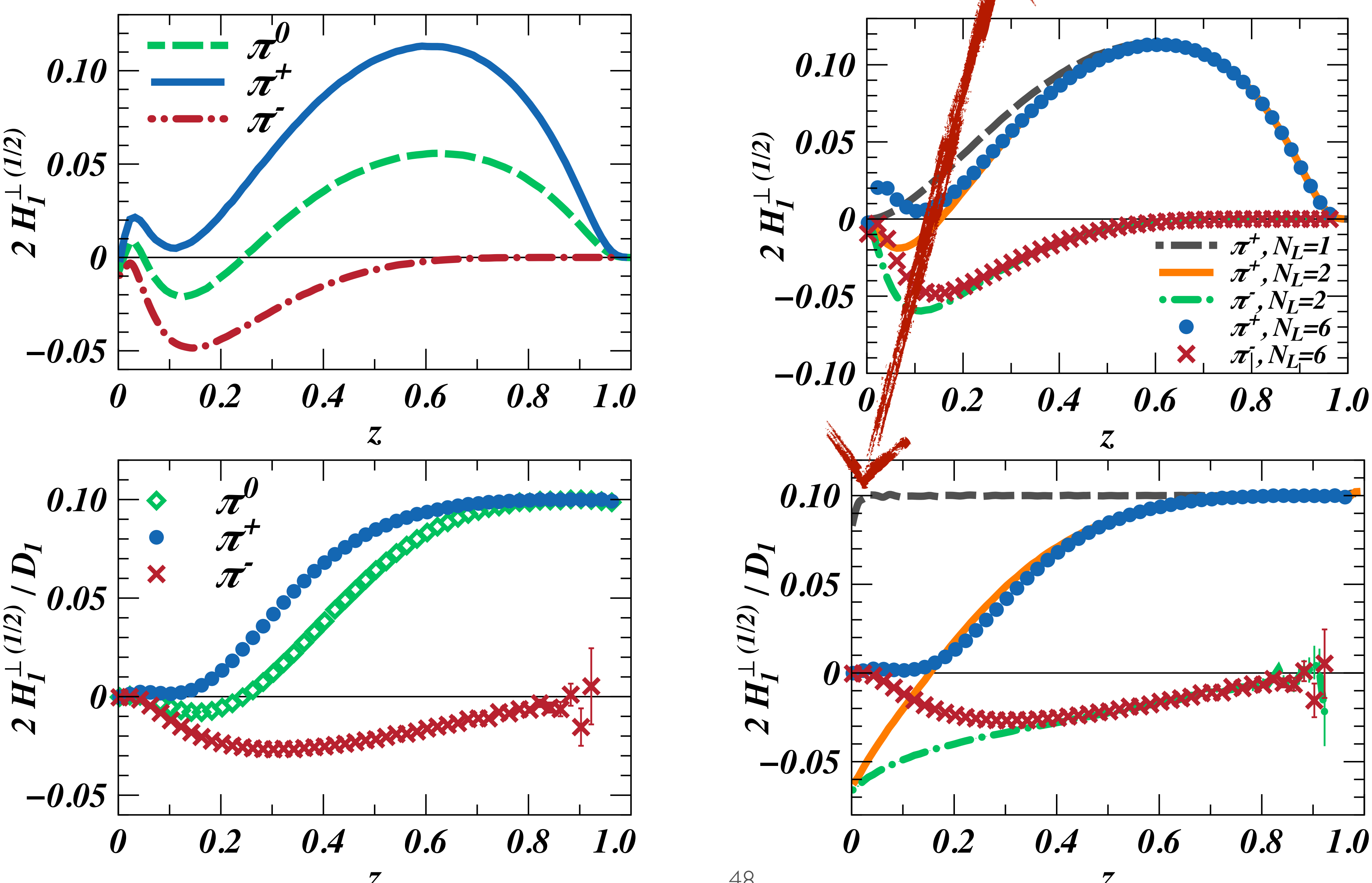


TOY MODEL RESULTS

$$d^{(toy)}_{h/q^\uparrow}(z,p_\perp^2) = d_1^{h/q}(z,p_\perp^2)(1+0.1\sin\varphi)$$

$N_{Links} = 6$

N_{Links} *Dependence*

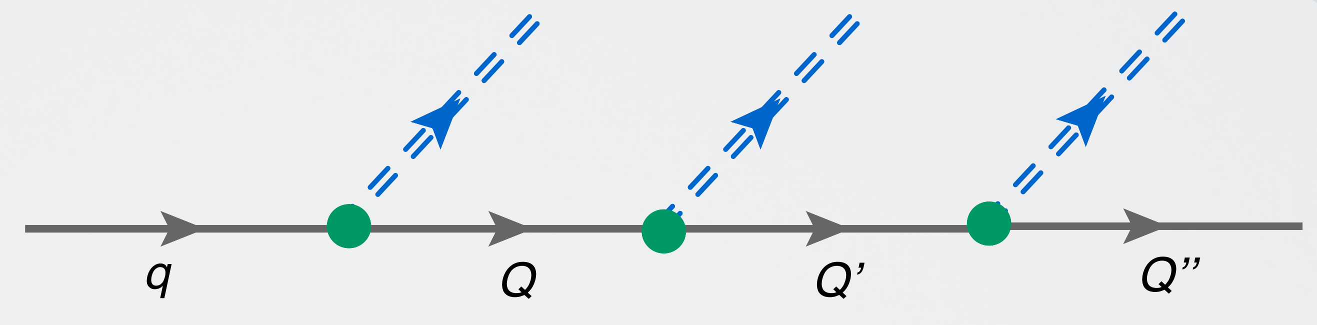


THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



The probability of finding mesons m with mom. fraction z in a jet of quark q

$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m\left(\frac{z}{y}\right)\frac{dz}{y}$$

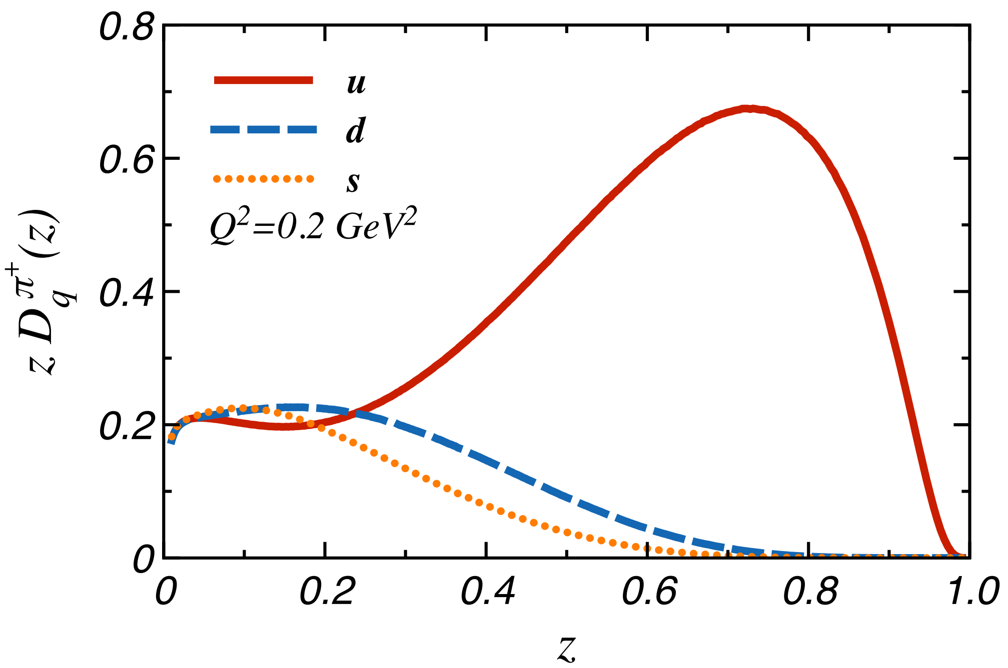
Probability of emitting the meson at link 1

Probability of Momentum fraction y is transferred to jet at step 1

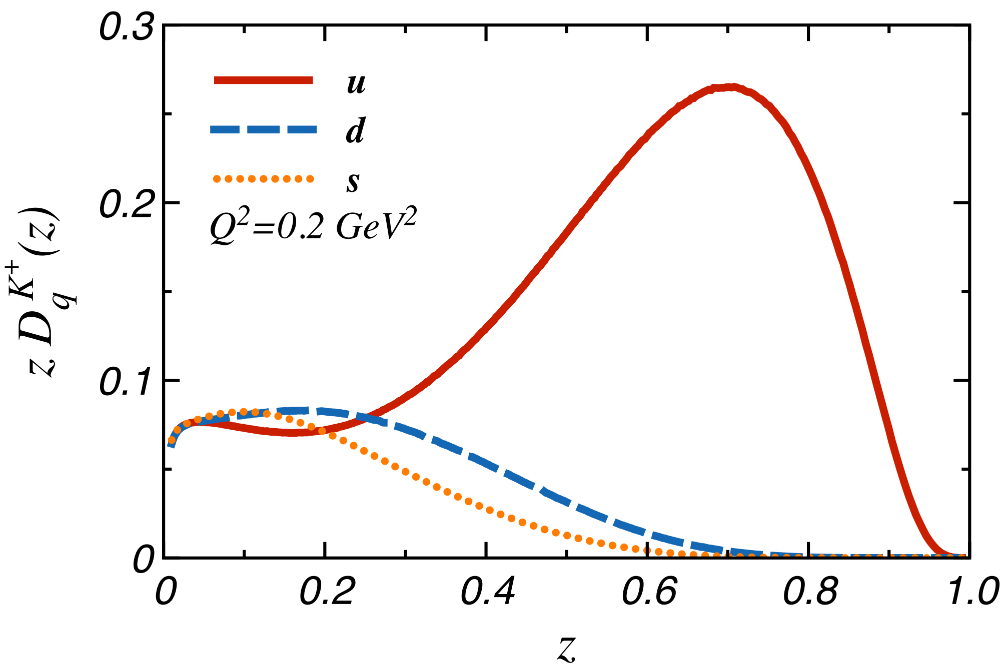
The probability scales with mom. fraction

SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011



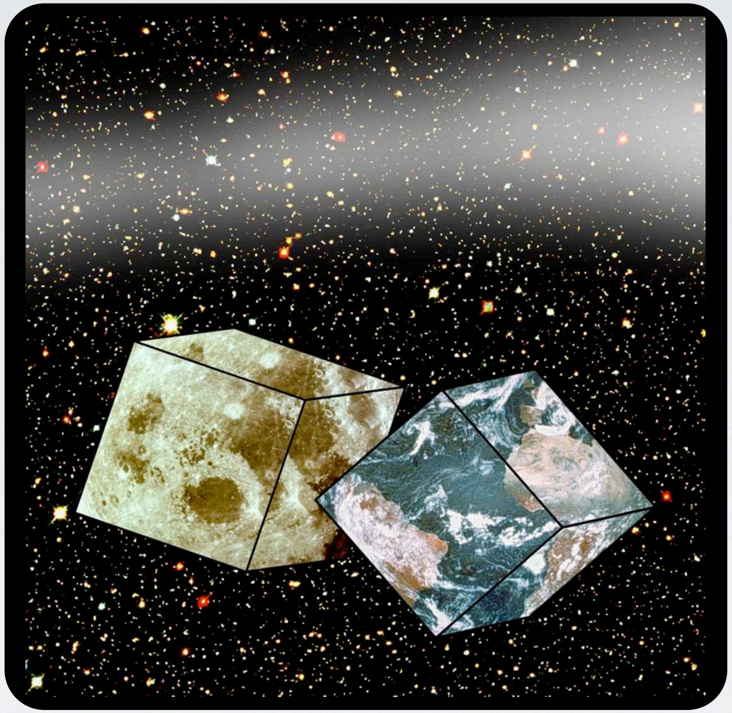
π+



K+

MONTE-CARLO (MC) APPROACH

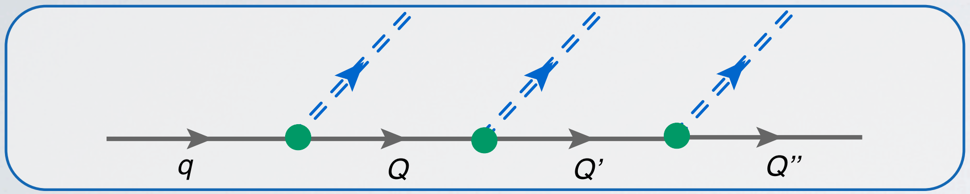
H.M., Thomas, Bentz, PRD.83:114010, 2011



- Simulate decay chains to extract number densities.
- Allows for inclusion of TMD and experimental cut-offs.
- Numerically trivially parallelizeable (MPI, GPGPU).

FRAGMENTATIONS FROM MC

- Assume Cascade process:



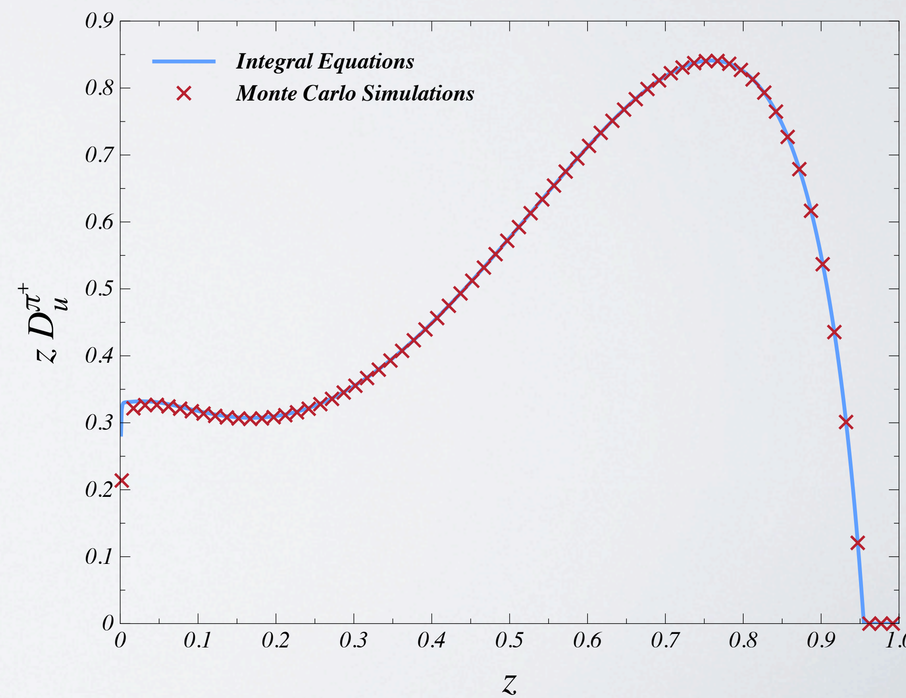
$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

- Sample the emitted hadron according to splitting weight.

- Randomly sample z from input splittings.

- Evolve to sufficiently large number of decay links.

- Repeat for decay chains with the same initial quark.

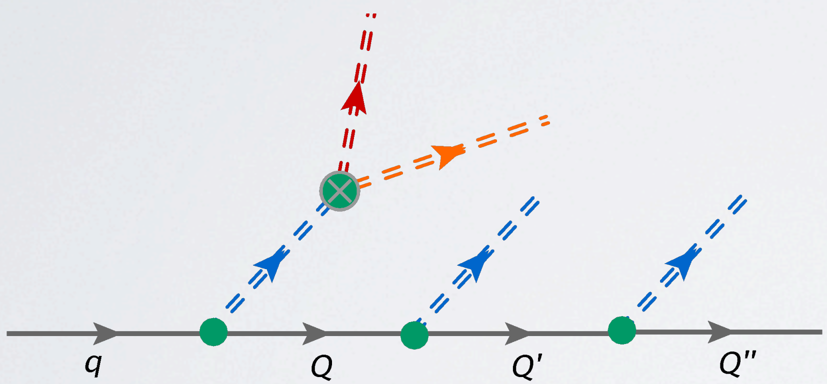


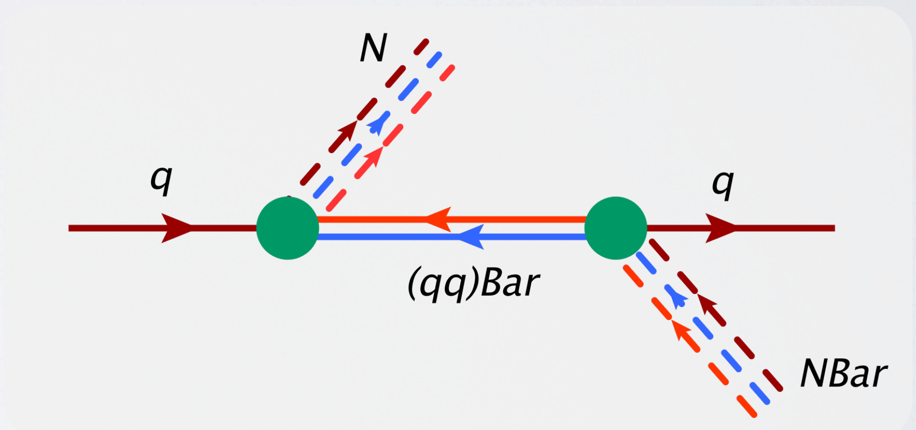
MORE CHANNELS

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$\boxed{h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}}$$

- Add the decay of the resonances:

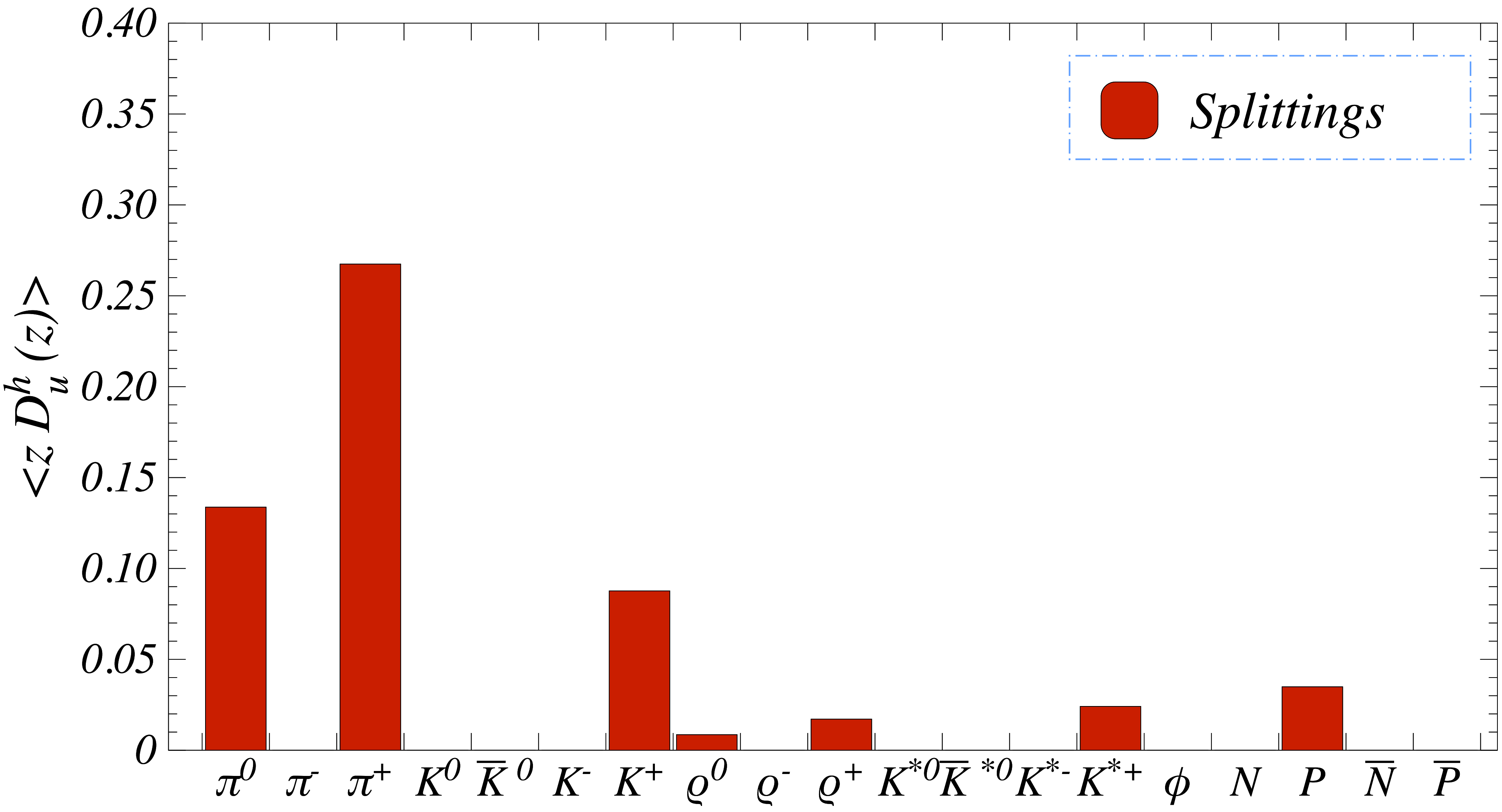




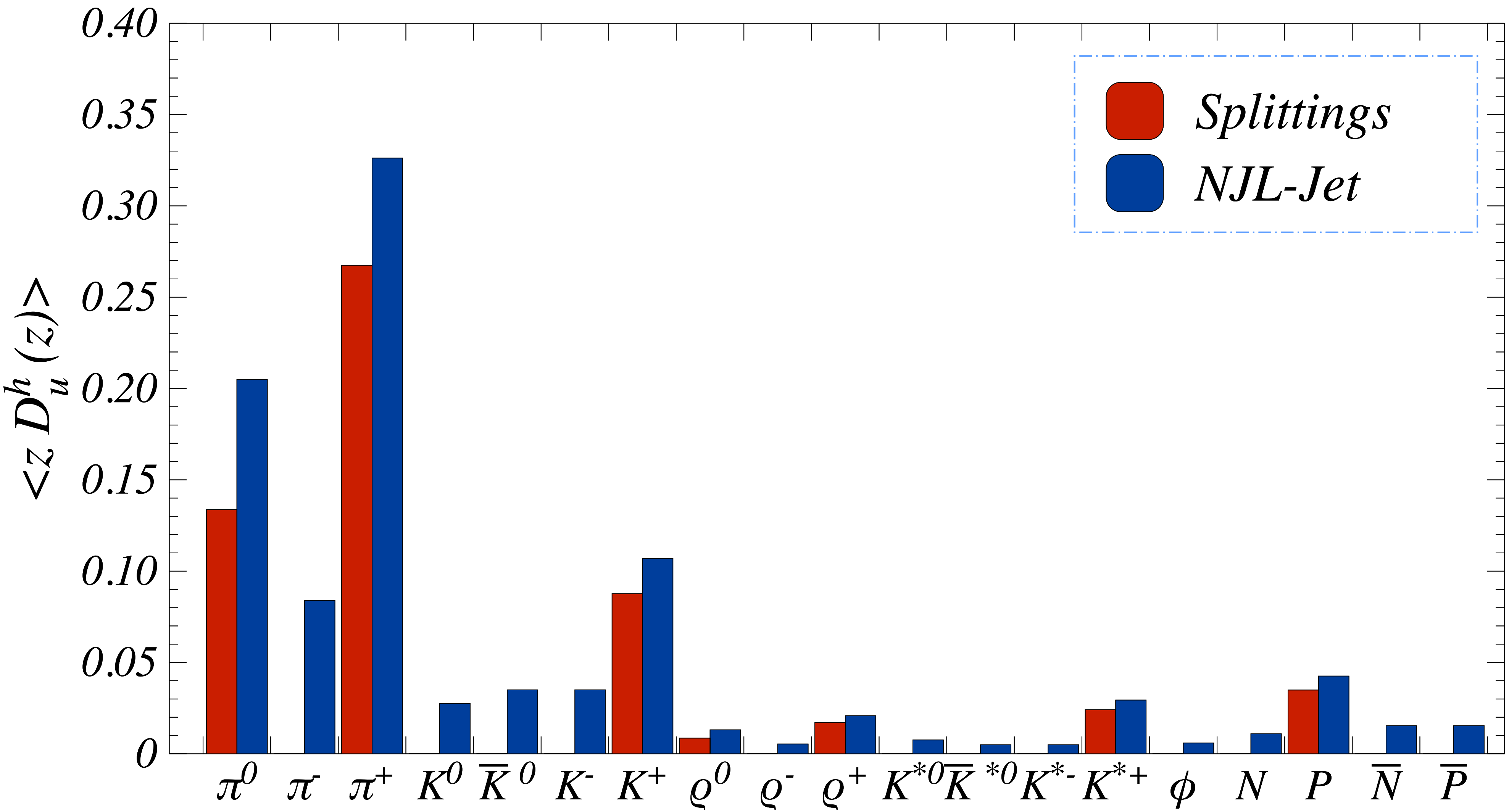
- Decay cross-section in light-front variables:

$$\boxed{dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\ m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}}$$

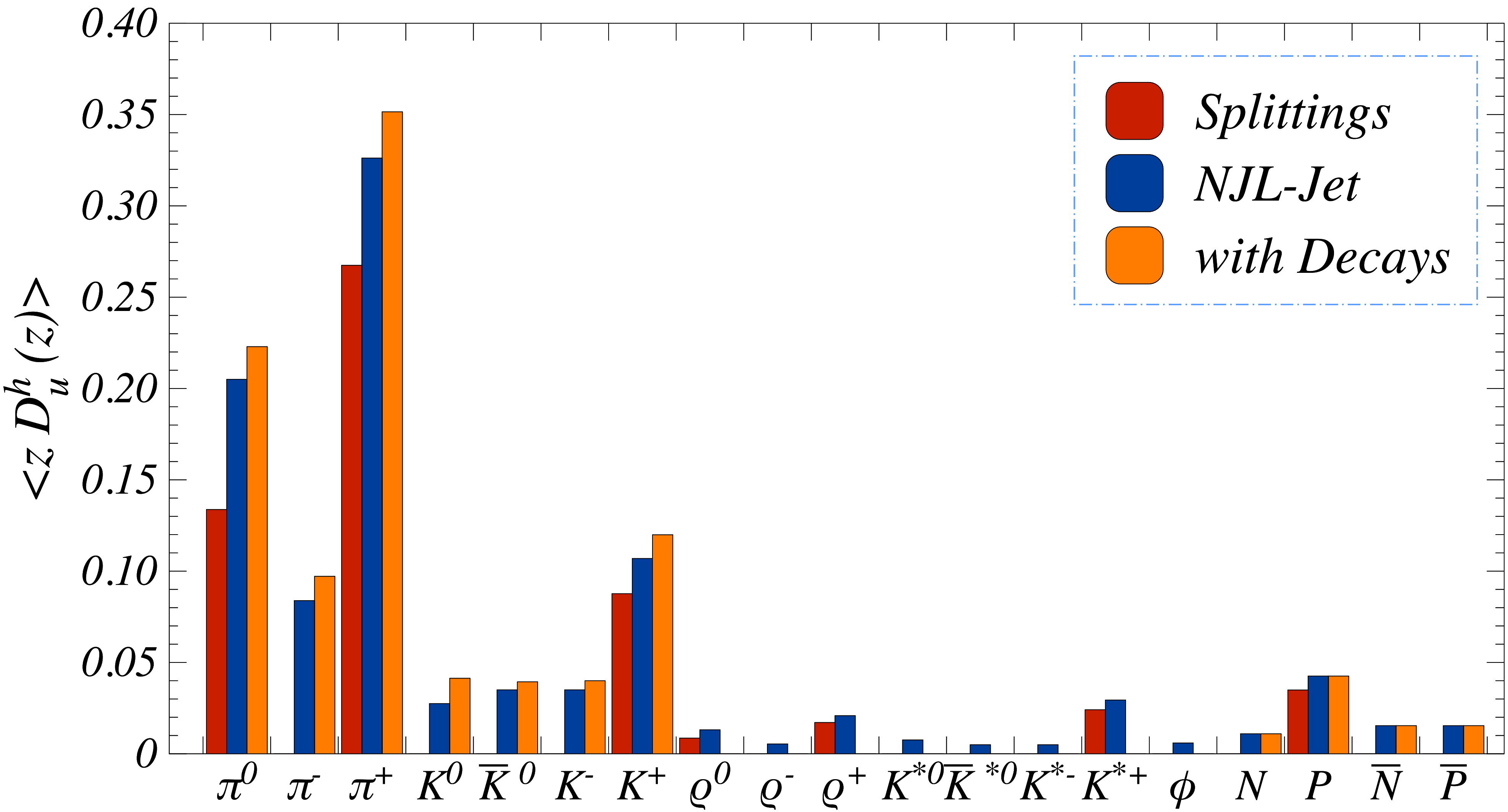
Results: Momentum Fractions



Results: Momentum Fractions



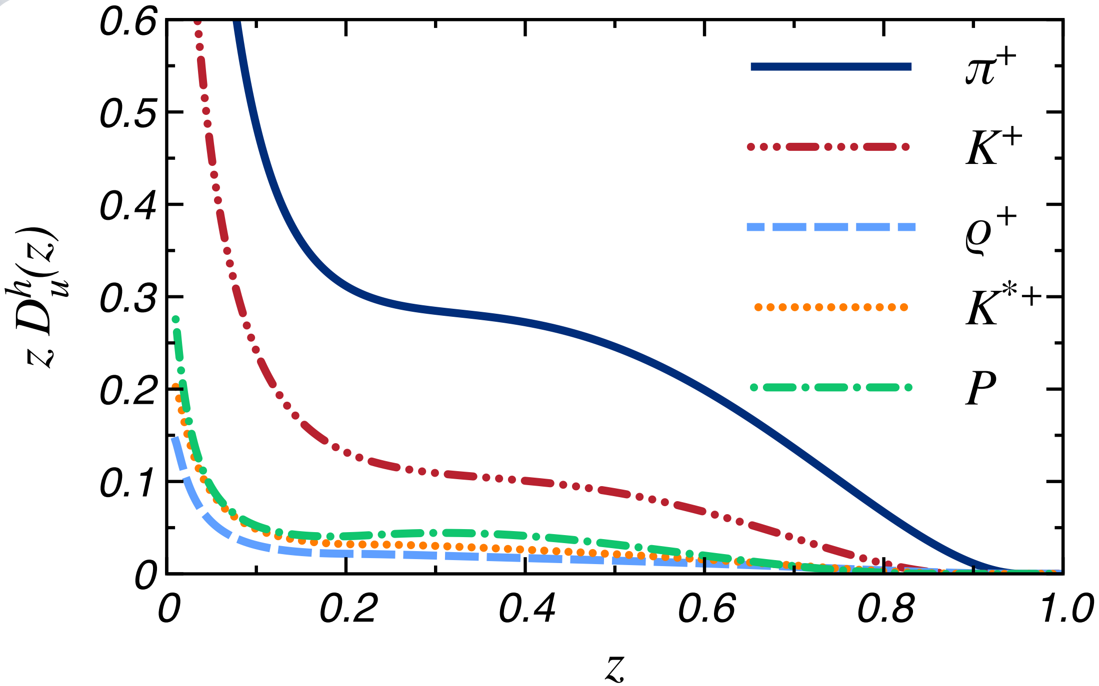
Results: Momentum Fractions



The Momentum (and Isospin) sum rules satisfied within numerical precision (less than 0.1 %)!

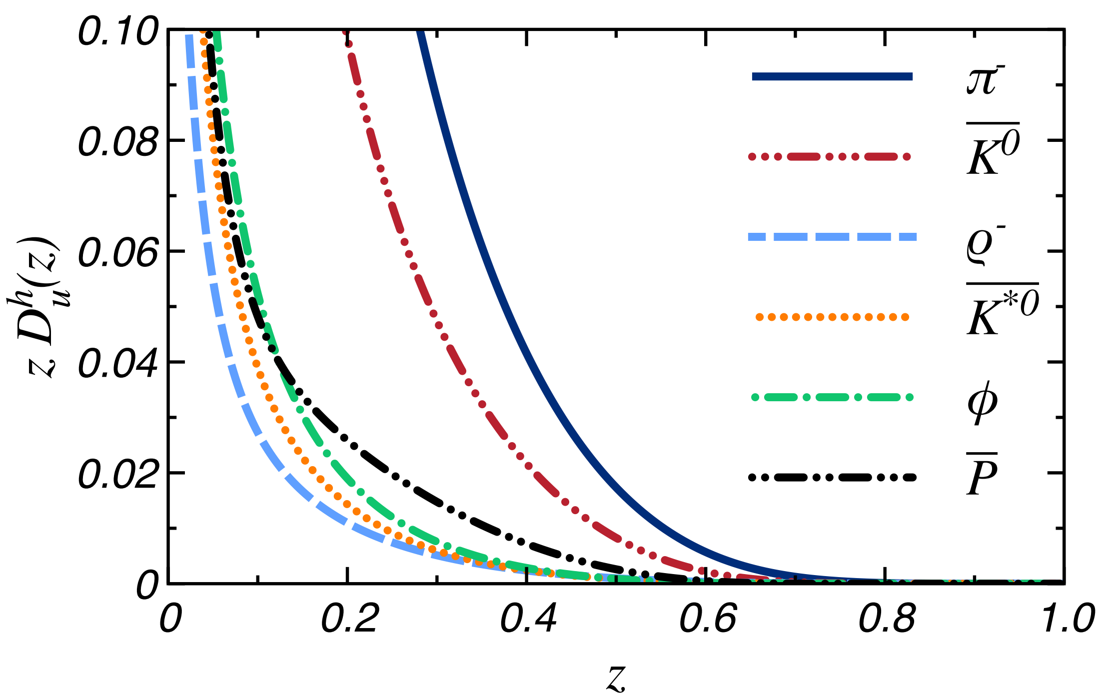
Results: Fragmentations to All Hadrons

$Q^2 = 4\ \mathrm{GeV}^2$



Favored

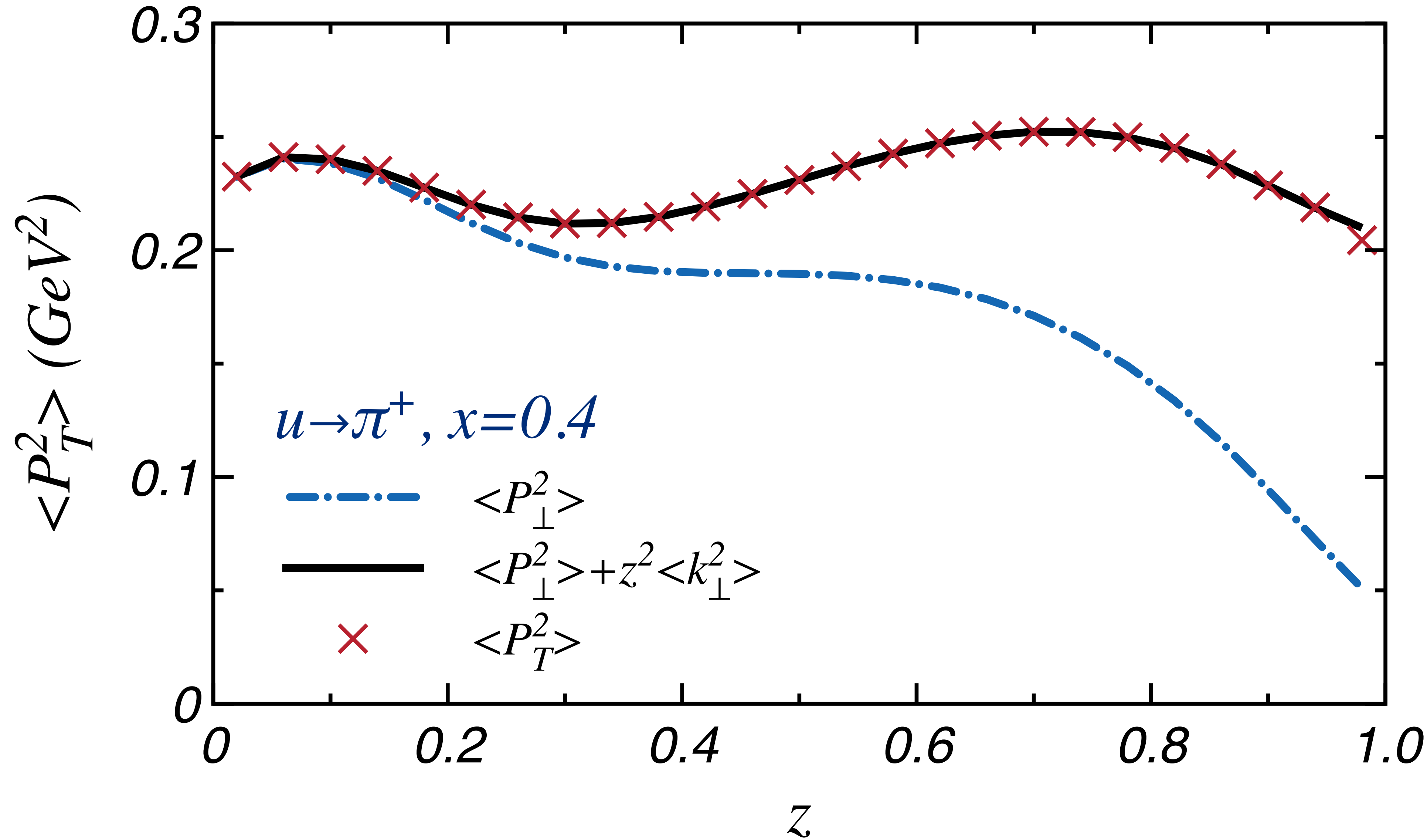
Unfavored



CROSS-CHECK OF MC FRAMEWORK

Input: $\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$

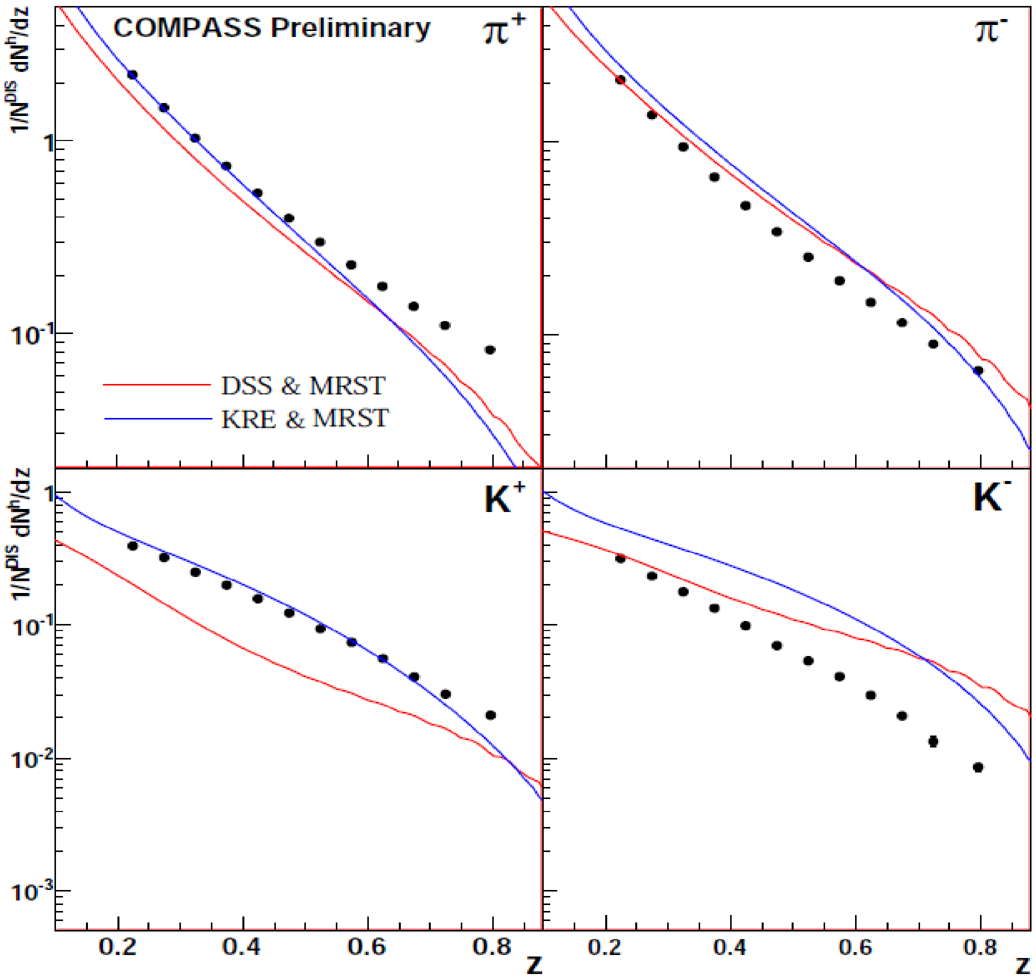
Output: $\langle P_T^2 \rangle(x,z) = \langle P_\perp^2 \rangle(z) + z^2 \langle k_T^2 \rangle(x)$



Unfavored FFs NOT well known!

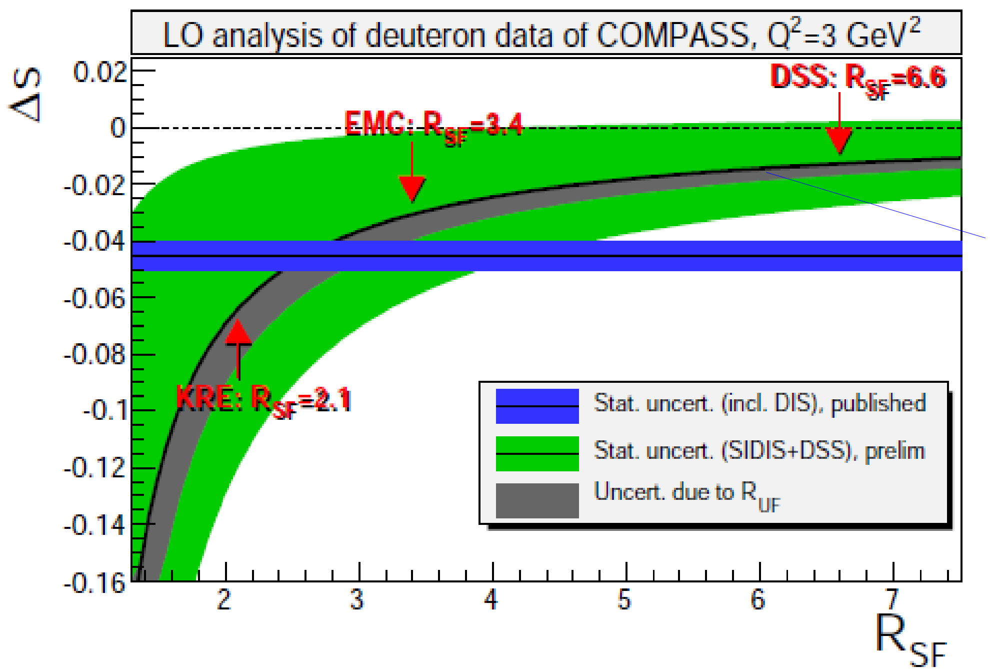
From talk by Celso Franco for COMPASS at CIPANP 2012

Hadron Multiplicities (Preliminary)



Impact on Extraction of Δs

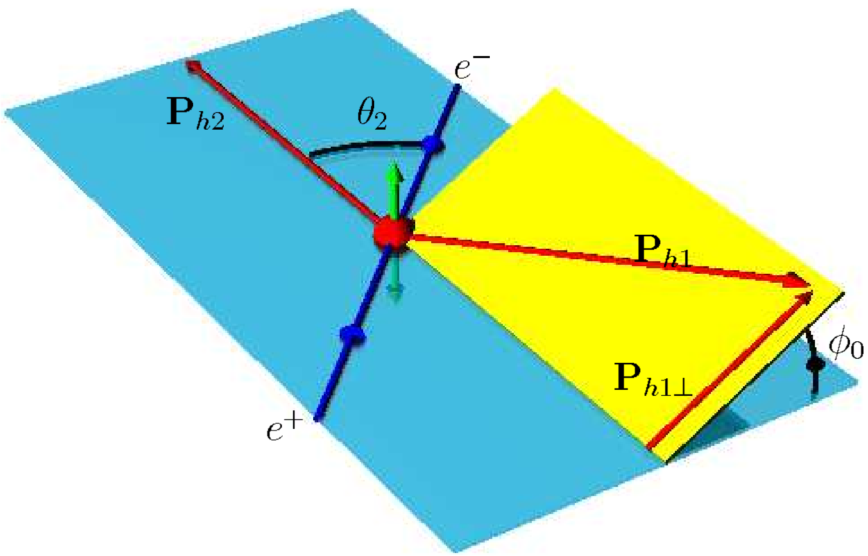
$$R_{UF} = \frac{\int_{0.2}^{0.85} D_d^{K^+}(z)\,dz}{\int_{0.2}^{0.85} D_u^{K^+}(z)\,dz}, \quad R_{SF} = \frac{\int_{0.2}^{0.85} D_{\bar{s}}^{K^+}(z)\,dz}{\int_{0.2}^{0.85} D_u^{K^+}(z)\,dz}$$

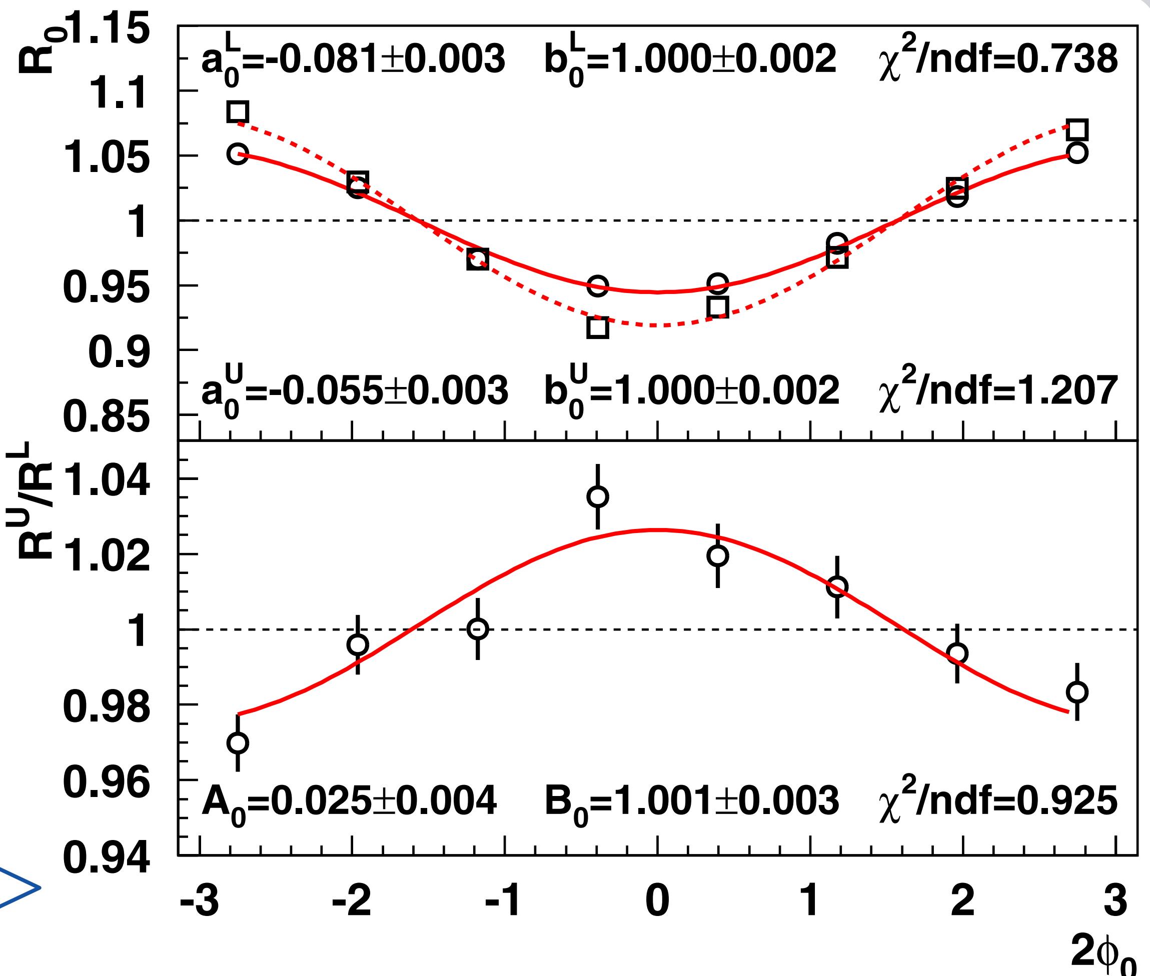


DIRECT EXPERIMENTAL CONFIRMATION

BELLE, R. Seidl et al.,Phys.Rev.Lett. 96, 232002 (2006).

$$e^+e^- \to h_1 h_2 X$$





$$R_0 = N_0(2\phi_0)/< N_0 >$$

$$\boxed{R_0 = a_0 \cos(2\phi_0) + b_0}$$

$$a_0(\theta_2, z_1, z_2) = \frac{\sin^2\theta}{1+\cos^2\theta} \frac{f(H_{1,q}^{\perp}(z_1) H_{1,\bar{q}}^{\perp}(z_2))}{D_1^q(z_1) D_1^{\bar{q}}(z_2)}$$